# MUSICAL PITCH

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# MEASUREMENT OF INTERVALS

# AMONG THE ANCIENT GREEKS.

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The material available for reconstructing the music of antiquity is unfortunately very meagre. For the study of ancient sculpture, architecture, poetry, and painting (using the word in its broadest sense)the modern world has only to turn to the existing monuments of ancient activity in each of these fine arts. The fullest appreciation of the art of any former age can only be gained by the contemplation of actual artistic creations left by the artists themselves. How little of ancient sculpture or architecture should be known, if our knowledge were derived solely from the works of contemporaneous writers, however excellent! The study of ancient music, although we are not so badly off in it as we should be in sculpture or architecture in such a case, is at present in a condition which resembles in a measure such a state of affairs. Suppose all architectural remains of the Greek race were utterly lost, but that ancient critics had left us not only descriptive matter, but actual plans of temples, dwellings, and so

forth. In proportion as these plans were perfect, we should be in a position at any moment to construct more or less accurate representations of these ancient buildings or even to make a life-size restoration. In such a case ancient architecture would be in a condition, so far as our appreciation of it is concerned, analogous to that in which ancient music now finds itself. We have admirable theoretical treatises and we have also a few incomplete plans and specifications; for what is a musical score but a drawing or a ground plan of the musical structure? To reproduce it, all that is needed is a knowledge of the symbols employed in the specifications and the means to interpret them according to the conventions there used.

It will thus be seen that Music differs from the great space-arts in the transitory nature of its material, sound, and in the consequent necessity for fresh representations whenever it is desired actually to realize any of its creations.

Poetry, the fifth of the five greater Fine Arts, although it is classed with Music as a time art, is in a vastly better position than Music for perpetuating its productions. To take the case of Greek Poetry, we are able to lay down with no little confidence rules for the correct pronunciation of

the words and for the correct scansion of the metre. But even if by some supernatural means we should learn that all our suppositions on these points are utterly wrong, we should still possess the major part of what constitutes Poetry, the thought, which is imperishable. The written word is in Poetry not a bad substitute for the spoken word, whether centuries or only hours have passed since it was written. And when, as is probably the case, all the essential features of the pronunciation are in our possession, there is nothing lacking which is actually indispensable to our enjoyment. But with Music there is no other element, besides the sounds themselves, which has any claim to the name of music at all. While rhythm is perhaps, after the sounds, the most important constituent, rhythm alone is not music.

The problem presented to the modern musical antiquarian is how to reconstruct the sounds of ancient music from the data furnished by the ancients.

For the solution of the question in any given case, as, for instance, in the case of the Delphian Hymn to Apollo, it is necessary to determine some four elements, the melody, the harmony ( in modern sense ), if there be any, the time or rhythm, and the instrumentation, including under this term

all that concerns the timbre or quality of the sounds, and their force or the modulations of loud and soft.

In the case of the Hymn mentioned, as for the instrument, we know that the music was sung. The determination of the pronunciation may be referred to other studies. Howsoever settled, the question would not materially affect the music proper. The right degrees of loudness and softness and the quality or color of the voices may safely be left to our own judgment. As to harmony ( in the technical sense ) among the voices, we know that there was none; that which certainly existed between the voices and the accompanying instruments, and, the quality of the tones of the instruments, are lost to us. The rhythm is preserved in the words to which the music was sung. The rest of what makes up the time, namely, the speed with which the ode should be played, is of minor importance. There remains for determination, the melody or the tune itself.

In like manner the few other pieces of ancient music would have to have these various elements determined in some way or another.

But in all of them the most important and the most difficult determination is undoubtedly that of the melody.

In seeking to find equivalents for the written symbols by which any music is expressed the first step must be to discover the order or succession of the notes in the matter of acuteness and graveness of pitch, and the next must be to ascertain the exact distances at which the notes stand with reference to one another.

The invaluable work of Alypius enables us to lay down with certainty the order of the notes in the Greek notation. By means of this information alone we could plot a curve of any melody, such that every rise and fall in pitch was represented, and only the amount of this motion would remain undetermined. All the measurements are wanting and the outline is seen in a distorted perspective. The second step supplies this deficiency. We ascertain for each note the distance or interval at which it stands from its neighbors. This done, we are in possession of all the <u>knowledge</u> requisite to reproduce the notes of any notation. Our ability to translate the notation correctly into modern notation and our ability to reproduce the sounds vocally or instrumentally, will, of course, depend for the most part on the similarity of the music in question with modern music.

The present dissertation concerns itself with this second step and is an attempt to show to what extent our knowledge of the absolute width of musical intervals in general among the Greeks is based on firm foundations. The extant fragments of ancient music themselves and the notations in which they are written are not discussed. It is not sought to find modern equivalents for the actual notes nor for the intervals occurring in any of the ancient scales or systems. The subject is the measurement of intervals abstractly considered, irrespective of their place in the scale, and irrespective of their function in actual representation. Pitch is consequently considered not as a quality but as a quantity. It is not the position of notes on the scale of absolute acuteness and graveness, nor is it their relative positions as members of a collection or series of which we treat, but only their mutual relation as to interval apart from position and function. Therefore no discussion is made of the absolute width of the Greek musical scale as it varied from time to time with the advance of music to an independent place among the arts, nor of the absolute pitch of Greek notes; nor are the various scales touched on except in so far as they throw

light on the width of certain intervals.

The ancient explanation of sound as a physical phenomenon, however insufficient from a modern point of view, is accurate enough in general for musical purposes. The important part played by the air either as the cause of sound, or as the medium of its transmission to our ears, seems to have been generally recognized. Aristotle knew that air suffered condensation and rarefaction in the production of sound, if this is his meaning when he says that sound is not caused by the air taking on itself a shape or form, as some hold, but by its being moved through contraction and stretching.

Aristotle, De Audib. p. 800, a. l. 1, Ed. Bekker.

τὰς δὲ φωνὰς ἁπάσας συμβαίνει γίγνεσθαι καὶ τοὺς ψόφους ἢ τῶν σωμάτων ἢ τοῦ ἀέρος πρὸς τὰ σώματα προσπίπτοντος, οὐ τῷ τὸν ἀέρα σχηματίζεσθαι, καθάπερ οἴονταί τινες, ἀλλὰ τῷ κινεῖσθαι παραπλησίως αὐτὸν συστελλόμενον καὶ ἐκτεινόμενον καὶ καταλαμβανόμενον, ἔτι δὲ συγκρούοντα διὰ τὰς τοῦ

πνεύματος καὶ τῶν χορδῶν γινομένας πληγάς· ὅταν γὰρ τὸν ἐφεξῆς ἀέρα πλήξῃ τὸ πνεῦμα τὸ ἐμπίπτον αὐτῷ, ὁ ἀὴρ ἤδη φέρεται βίᾳ, τὸν ἐχόμενον αὐτοῦ προωθῶν ὁμοίως, ὥστε πάντῃ τὴν φωνὴν διατείνειν τὴν αὐτὴν, ἐφ' ὅσον συμβαίνει γίνεσθαι καὶ τοῦ ἀέρος τὴν κίνησιν.

The last sentence seems to imply an actual transference of the air itself. So do the words: ὁ γὰρ ἀθρόος ὑπὸ τῆς πληγῆς ἀὴρ μέχρι μέν τινος φέρεται συνεχής. (p. 801,a,l.24), Again in discussing the effect of speaking through tubes, Aristotle says ( p. 801, a, 29 ) πᾶσαι δοκοῦσιν αἱ φωναὶ ἀκεραίως εἶναι πλησίον τῆς ἀκοῆς διὰ τὸ μὴ σκεδάννυσθαι τὸν ἀέρα φερόμενον.

The view that sound is a formation of the air is found in the Problems in the section Περὶ Φωνῆς. Problems XI 23 διὰ τί, εἴπερ ἡ φωνή ἐστιν ἀήρ τις ἐσχηματισμένος καὶ φερόμενος, διαλύεται πολλάκις τὸ σχῆμα, ἡ δὲ ἠχὼ κτἑ.
XI 51 διὰ τί, εἴπερ ἡ φωνὴ ἀήρ τις ἐσχηματισμένη, ἔτι, φερομένη διαλύεται πολλάκις τὸ σχῆμα, ἡ δὲ ἠχὼ κτἑ.

But the other definition is also found

XI 35 ἡ δὲ φωνὴ κίνησίς ἐστι.

Compare too

XI 29 ὁ δὲ ψόφος ἀὴρ ἢ πάθος ἀέρος ἐστίν.

XIX 35 ἡ δὲ φωνὴ ἢ ἀέρος ἢ ἄλλου τινὸς φορά.

The following definitions contain the idea of a blow on, or an impulse imparted to the air.

Adrastus in Theo Smyrnaeus De Mus. c. 6, p. 50, Ed. Hiller.

φησὶ δὲ καὶ τοὺς Πυθαγορικοὺς περὶ κανόνων εἶναι τεχνολογεῖν· ἐπεὶ μέλος μὲν πᾶν καὶ πᾶς φθόγγος φωνή τίς ἐστι, ἅπασα δὲ φωνὴ ψόφος, ψόφος δὲ πλῆξις ἀέρος κεκωλυμένου θρύπτεσθαι κτέ.

(Quoted also by Bryennius, p. 394, Ed. Wallis.)

Nicomachus, Harmonices Manuale, p. 7 Meib.

καθόλου γάρ φαμεν ψόφον μὲν εἶναι πλῆξιν ἀέρος ἄθρυπτον μέχρι ἀκοῆς.

Plutarch, Conviviales Disputationes VIII, iii, p. 975

ἡ δὲ φωνὴ πληγὴ σώματος διηχοῦς· διηχὲς δὲ τὸ συμπαθὲς αὑτῷ καὶ συμφυές, εὐκί-

γητον δὲ καὶ κοῦφον καὶ ὁμαλὸν καὶ ὑπήκοον τοῦ δὲ εὐτονίαν καὶ συνέχειαν οἷός ἐστι παρ' ἡμῖν ὁ ἀήρ.

Plutarch, de Placitis Philosophorum ( where a number of definitions are collected.)

IV. xix. Περὶ Φωνῆς, 902 B. fol.

Πλάτων τὴν φωνὴν ὁρίζεται, πνεῦμα διὰ στόματος ἀπὸ διανοίας ἠγμένον, καὶ πληγὴν ὑπὸ ἀέρος δι' ὤτων καὶ ἐγκεφάλου καὶ αἵματος μέχρι ψυχῆς διαδιδομένην.

Ἐπίκουρος, τὴν φωνὴν εἶναι ῥεῦμα ἐκπεμπόμενον ἀπὸ τῶν φωνούντων, ἢ ἠχούντων, ἢ ψοφούντων· τοῦτο δὲ τὸ ῥεῦμα εἰς ὁμοιοσχήμονα θρύπτεσθαι θραύσματα.

Δημόκριτος, καὶ τὸν ἀέρα φησὶν εἰς ὁμοιοσχήμονα θρύπτεσθαι σώματα καὶ συγκαλινδεῖσθαι τοῖς ἐκ τῆς φωνῆς θραύσμασι.

IV xx. Εἰ ἀσώματος ἡ φωνή, καὶ πῶς ἠχὼ γίνεται.
902 F. fol. Πυθαγόρας, Πλάτων, Ἀριστοτέλης ἀσώματον. οὐ γὰρ τὸν ἀέρα, ἀλλὰ τὸ σχῆμα τὸ περὶ τὸν ἀέρα καὶ τὴν ἐπιφάνειαν, κατὰ ποιὰν πλῆξιν γίνεσθαι φωνήν. πᾶσα δὲ ἐπιφάνεια ἀσώματος.
Οἱ δὲ Στωϊκοὶ, σῶμα τὴν φωνήν.

Claudius Ptolemy's definition is as follows:

Harmonics I. iii. p. 6, Ed. Wallis.

τάσις γάρ τις ἐστι συνεχὴς τοῦ ἀέρος, ὁ ψόφος, ἀπὸ τοῦ τοῖς τὰς πληγὰς ποιοῦσιν ἐμπεριλαμβανομένου διήκουσα πρὸς τὸν ἐκτός.

Cp. Harm. I. i. p. 1, ψόφος δὲ πάθος ἀκουστόν.

See too Bryennius I. Sect. iv. p. 377, Ed. Wallis.

Aristides Quintilianus, De Musica, p. 7, l. 7, Ed. Meib:

ὕλη δὲ μουσικῆς φωνὴ καὶ κίνησις σώματος. τὴν δὲ φωνὴν οἱ μὲν ἀέρα πεπληγμένον, οἱ δὲ ἀέρος πληγὴν ἔφασαν, οἱ μέν, αὐτὸ τὸ σῶμα τὸ πεπονθός, ἦχον, οἱ δ', ὅπερ ἄμεινον, τὸ τούτου πάθος, ὁριζόμενοι.

Sounds differ from one another in a number of ways. To mention a few of these, a sound may differ from another in loudness, in duration, in timbre or quality, or in pitch. In order to define a sound completely, it would be necessary to specify a great number of particulars. It would be necessary to state whether it was articulate or inarticulate, compound or simple, both in respect of its complexity at any given time, and in respect of its variability from time to time during its existence, whether it was musical or unmusical, and, if it has that quality known as pitch, it would be necessary to give the degree of pitch, or degrees of pitch, if more than one. If the sound occurred in a piece of music, it would be necessary to give a number of additional data, such as its relative duration, comparing it with the context, its position in the bar, its quality or timbre and its intensity or force. If the sound were articulate ( and it might be articulate in addition to being musical ) it would involve an analysis into its phonetic elements, and a statement of the loudness or softness and of the pitch at which it is spoken or sung.

For a complete analysis and classification of sounds, it

would be necessary to find an answer to the question, What is the most elementary sound imaginable? or at least to select provisionally sounds of such a nature that they cannot readily be separated into parts. For speech the elements have been analyzed from early times. They are the vowels and consonants. This must have been done simultaneously with the origin of writing by means of letters - that is, when writing ceased to be ideographic. In the case of Music, there is no difficulty in making a time analysis of the elements. The notes themselves correspond to the letters of a word, and the kind of sound is indicated more or less perfectly according to the notation. The quality of pitch is, of course, indicated as accurately as is practicable. Other qualities are indicated by marks and words. The analysis of sounds into coexisting elements is a different matter, and has been solved only in this century by the discovery of Helmholtz that the quality or timbre of a note is the result of compounding 'pure tones' according to certain principles.

But, however successful the scientific attempt to find the elements of sounds may be, each science whose material is sound, as musical science, phonetics, metrical science, will make its own classification of sounds for its own sphere.

For Phonetics, the distinction between articulate and inarticulate sounds is of the first importance. For Music the distinction between musical and unmusical sounds is naturally of equal importance for the subject, and it is usual in books on musical theory to start with this distinction.

What is meant by musical and unmusical ( or non-musical)? If by 'musical' we mean 'used in music,' the distinction only involves an enumeration of the sounds so used. If not, we must find out what it is that makes a sound musical. What is the basis for the classification? Unfortunately, if we make an examination of the definitions of the 'musical sound', we shall find that the basis is not always the same. Without a clear statement of the principle on which this classification is made, we run the risk of propounding the truism alluded to, that musical sounds are sounds that are musical.

The distinction between musical and non-musical sounds in its main features is readily grasped. It is the difference between the speaking voice and the singing voice, between the howling of the wind and the notes of a flute. In extreme cases there will be no difficulty in classifying a sound under one or the other of these categories. In intermediate cases it will sometimes be difficult to decide whether a

given sound has more the character of a musical note or that of a noise. A sound may be partly musical and partly unmusical, and yet the parts may be intimately connected, or even inseparable in thought.

But in passing from sounds which are purely musical through all degrees to sounds which are purely noisy, it will be noticed that there is a gradual disappearance of a certain element. This element is pitch, and on it in the last analysis rest all the various manners of distinguishing musical sounds from non-musical sounds. All musical sounds, in fact, nearly all the sounds used in music of whatever nature (the few exceptions are furnished by instruments, like cymbals, castanets, etc., which are used chiefly for rhythmic effects), are characterized by the presence of this quality of pitch. We might make the first grand division of sounds depend on the presence or absence of this property.. To be sure this would not give us an infallible test for determining musical sounds, nor could we draw a hard and fast line, owing to the fact that pitch may be present in varying quantities. But at least we could separate sounds in which it is impossible to recognize any trace of pitch from those in which it is

present to some extent. After that the steadiness or unsteadiness of pitch would furnish a means of making a subdivision which would correspond to the distinction between musical and unmusical sounds as it is often drawn. For example, these sounds are defined in Sedley Taylor's Sight Singing from the Established Notation in the words, § 6, "A musical sound is one of <u>constant</u>, a non-musical sound one of <u>varying</u>, pitch." (Cf. Sec. 3, and his Sound and Music Sec. 23.) But this classification, on the one hand, excludes from the category of musical sounds that union of two notes executed on the violin or related instrument or sung by the voice, known as portamento, and, on the other, admits sounds such as those of badly made bells, in which the pitch, such as it is, may remain constant and steady.

This leads one to another principle which might be used for a subdivision of pitch-sounds. The first was the behavior of pitch from moment to moment, its changeability or constancy. The one now suggested is the complexity or simplicity of the arrangement of the various heights or degrees of pitch which are present at any and every point of time, in every sound except the theoretically pure tone of science. If the constituent pitches are arranged with reference to some

principle of order, the sound will be musical, and on this order will depend the quality or timbre of the note. On the other hand, if the arrangement is disorderly, the sound will be unmusical or a noise. A bell may have been so unsuccessfully cast, that, apart from the variation so noticeable from time to time, the different parts give forth sounds of different and unrelated pitches. The result is a noise. If all the keys of a piano which one can cover with the hand are sounded at once, a noise is produced.

This principle of order, if introduced into the classification first noticed ( based on the steadiness of the pitch), would subdivide those sounds whose pitch was unsteady or varying into two classes, of which the orderly class would embrace sounds which are admittedly musical, (portamento), but are often excluded. In this subdivision the orderly variation in pitch is in the control of the artist ( the violinist or singer). In the class in which the pitch is steady, the orderly disposition of the constituent pitches ( the overtones) is furnished by nature, and varies according to the instrument ( and is the cause of the timbre.)

Pitch is that quality wherein the sounds produced by the

same instrument and possessing the same degree of loudness differ from one another. Omitting from consideration the element of time, we may define a musical sound by giving its loudness, its quality, and its pitch. We may compare these three manners in which sounds may differ with the three dimensions necessary and sufficient for the definition of a rectangular solid. To express degrees of pitch, pairs of adjectives, are used, as high and low, acute and grave, sharp and flat, shrill and deep. The majority of sounds ordinarily heard are complex in possessing a number of different, but often related, degrees of pitch, but in most musical sounds there is one degree which is stronger than the others, and the pitch of the sound as a whole is taken to be that of this most prominent element. Since there is no indication that the quality or timbre of sounds was analyzed in this way by any of the ancients, we shall not have to do with any but the nominal pitch of a sound.

The word pitch refers of course to height. The Greek words for pitch, τόνος and τάσις, are taken from the idea of stretching or straining. It would be observed from stringed instruments that variation in the tension of the

strings produced variation in the pitch. A similar state of affairs might easily be noticed in the vocal organs in singing. Of course the name for pitch might have been taken from the idea of the size or length of strings and of flutes. So far as it goes, the fact that τόνος and τάσις are derived from τείνω may be taken as evidence that the earliest stringed instruments had strings of equal size and length. Increase and relaxation of tension produce heightening and lowering of the pitch respectively. The Greek words ἐπίτασις and ἄνεσις are used for these operations. Height in pitch or high pitch is expressed by ὀξύτης, the reverse by βαρύτης, the adjectives ὀξύς and βαρύς signifying acute and grave respectively.

A rigorous definition of pitch does not seem to have been attempted in the ancient works on musical science. It was usual to define the term by means of ὀξύτης and βαρύτης, or by means of ἐπίτασις and ἄνεσις. Either the idea of height and depth in pitch was assumed and pitch itself was defined as that which is common to these, as was done by Claudius Ptolemy ( Harmonics, Bk. I., c. iv. p. 8,

Ed. Wallis - Opera Vol. III. -

ὁ γὰρ οὕτω λεγόμενος τόνος, κοινὸν ἂν εἴη γένος τῆς ὀξύτητος καὶ τῆς βαρύτητος, παρ' ἓν εἶδος τὸ τῆς τάσεως εἰλημμένος, ὡς τὸ πέρας τοῦ τέλους καὶ τῆς ἀρχῆς.

As 'limit' covers both 'end' and 'beginning', so τόνος both ὀξύτης and βαρύτης.

Compare Porphyry in his Commentary on Ptolemy's Harmonics - Ed. Wallis, p. 258: ἔστι γὰρ καὶ ἡ βαρύτης τάσις καὶ ἡ ὀξύτης τάσις );

or,

from the notion of change in pitch, as observed in the tones of the human voice and elsewhere, the concept of upward and downward motion was derived, and pitch was defined as the absence of such change or motion, that is as rest - ἠρεμία. This is the method employed by Aristoxenus. Τάσις is defined as μονή τις καὶ στάσις τῆς φωνῆς (Harmonic Bk. I. p. 12, Ed. Meibom.) Conversely ἐπίτασις (ἐπιτεῖναι) is defined by the help of τάσις: δεῖ δὲ καταμανθάνειν ὅτι τὸ μὲν ἐπιτείνειν τὴν

φωνὴ τὸ μέλεσιν ἐφ' αἷς τάσεως ἐστι (p. 13

One of the earliest definitions to be established in almost every treatise on the theory of music is the definition of the <u>musical sound</u>.

Aristoxenus set the example in this matter for a number of followers by preparing the way for this definition with a discussion of what is called κίνησις φωνῆς, <u>the motion of the voice</u>. He says ( p. 3, Meib.) that a description of the various kinds of κίνησις is necessary in order to define φθόγγος. Aristoxenus, Harmonic, I., p. 3: καὶ τοι τούτου μὴ διορισθέντος, οὐ πάνυ ῥᾴδιον εἰπεῖν περὶ φθόγγου τί ποτ' ἐστίν.

κίνησις φωνῆς is that motion by which one passes from a high note to a lower note, or from a low note to a higher note. It is, briefly, change of pitch. The ancients recognized two manners in which the pitch of the voice might move. Aristoxenus, p. 8, Ed. Meibom: πάσης δὲ φωνῆς δυναμένης κινεῖσθαι τὸν εἰρημένον τρόπον δύο τινές εἰσιν ἰδέαι κινήσεως, ἥ τε συνεχὴς καὶ ἡ διαστηματική.

It is evident that the passage from one degree of pitch to another must be made in one of two ways. Either the pitch of the sound changes suddenly from the initial state to the final state, so that at no moment does the sound rest at, or pass through, any intermediate degree of pitch between the extremes; or the pitch changes gradually in the direction of the final pitch, that is either upward or downward, and so passes through every possible intermediate degree, but rests at none of them. These are the only two ways in which a sound emanating from one and the same instrument can pass from one pitch to another. They may be compared respectively to stepping and gliding. In the one case the intermediate space is leapt over, in the other it is traversed. Now it might carelessly be supposed that since pitch is one-dimensional it would be impossible for a sound to alter its pitch without passing all intermediate degrees, unless there was an intervening moment of silence. Although in the actual production of such a change on a musical instrument capable of producing only one sound at a time, such as the voice, it is probably impossible to make this leap of pitch without a small moment of silence, it is not theoretically an impossibility as will be readily admitted when we consider that a new sound at the

new pitch may be started at any moment immediately before, at the exact instant of, or immediately after the cessation of the old pitch.

If now we consider the sequence of sounds emitted by any instrument and regard only the manner in which change of pitch takes place, it is plain that the former of the two manners, which we may call discontinuous, implies for practical purposes rest at various stages, that is to say, there will be a period of fixed pitch before and after each leap. We can of course conceive of a glide taking place immediately after a step, but such a performance would not be musical in any sense. The second manner, continuous change, implies nothing as to periods of fixed or stationary pitch. The glides may connect what are called notes or musical sounds as they are defined in the treatises ( a musical sound is one that has a constant pitch,) or there may be no such notes, the pitch may never become fixed, but may wander up and down at random. Connecting glides are denoted in music by the term <u>portamento</u>, and are familiar when the instrument is capable of this change of pitch. Such are the voice and bowed stringed instruments and some wind instruments. An example of aimless wandering of pitch is the howling of the wind in a storm.

We are now in a position to state accurately the ancient conception of the two kinds of motion. Κίνησις συνεχής is ( not that sort in which the alterations are continuous, but only) that sort in which both the alterations in pitch are continuous and there are no degrees of fixed pitch. Κίνησις διαστηματική on the other hand, is that method of moving in which the pitch leaps over intervals ( and so is called 'intervallate'),and then rests at various degrees of pitch. Aristoxenus is the best authority on the subject.

p. 8, Sec. 26, Ed. Meib:

κατὰ μὲν οὖν τὴν συνεχῆ, τόπον τινὰ διεξιέναι φαίνεται ἡ φωνὴ τῇ αἰσθήσει, οὕτως ὡς ἂν μηδαμοῦ ἱσταμένη ⟨ἢ⟩, μηδ' ἐπ' αὐτῶν τῶν περάτων, κατά γε τὴν τῆς αἰσθήσεως φαντασίαν, ἀλλὰ φερομένη συνεχῶς μέχρι σιωπῆς. κατὰ δὲ τὴν ἑτέραν, ἣν ὀνομάζομεν διαστηματικήν, ἐναντίως φαίνεται κινεῖσθαι, διαβαίνουσα γὰρ ἵστησιν αὑτὴν [αὐτὴν Codd.] ἐπὶ μιᾶς τάσεως, εἶτα πάλιν ἐφ' ἑτέρας, καὶ τοῦτο ποιοῦσα συνεχῶς — λέγω δὲ συνεχῶς κατὰ τὸν χρόνον — ὑπερβαίνουσα μὲν τοὺς περιεχομένους

ὑπὸ τῶν ἀκουόντων [illegible], ἱσταμένη δ' ἐπ' αὐτῶν τῶν τάσεων, καὶ φθεγγομένη ἑκάστας αὐτῶν ᾠ-τὰς μελῳδεῖν λέγεται, καὶ κινεῖσθαι διαστηματικὴν κίνησιν. And further on:-

p. 9 l. 11: ἁπλῶς γὰρ ὅταν οὕτω κινῆται ἡ φωνὴ ὥστε μηδαμοῦ δοκεῖν ἵστασθαι τῇ ἀκοῇ, συνεχῆ λέγομεν ταύτην τὴν κίνησιν. ὅταν δὲ στῆναί που δόξασα, εἶτα πάλιν διαβαίνειν τινὰ τόπον φανῇ, καὶ τοῦτο ποιήσασα, πάλιν ἐφ' ἑτέρας τάσεως στῆναι δόξῃ, καὶ τοῦτο ἐναλλὰξ ποιεῖν φαινομένη συνεχῶς διατελῇ, διαστηματικὴν τὴν τοιαύτην κίνησιν λέγομεν.

Why was the distinction of the kinds of motion regarded as of so great importance? It finds no place in modern works on the theory of music, unless it is implied in the definitions of musical and unmusical sounds.[1]

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1. Sedley Taylor Sounds and Music p. 48, 2nd Ed.:

"The difference, then, between musical and non-musical sounds seems to lie in this, that the former are constant while the latter are continually varying. The human voice can produce sounds of both classes. In singing a sustained note it remains quite steady, neither rising nor falling. Its conversational tone, on the other hand, is perpetually varying in height even within a single syllable; directly it ceas-

The reason for the great prominence given to this motion must lie in the relatively greater importance of the voice in ancient music, due more to the inferiority of the instruments than to any greater appreciation of the capabilities of the voice on the part of the ancients. Few at the present day consciously feel the changes in pitch which accompany the spoken sentence. The Greeks however were very sensitive to this element of speech. We have only to point to the fact that their written accents express this rise and fall of pitch. The limitations of this movement in point of acuteness and graveness are given by Dionysius of Halicarnassus in his treatise, De Compisitione Verborum. He says that the compass of

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es so to vary, its non-musical character disappears, and it becomes what is commonly called 'sing-song'. Compare the same author's Sight Singing from the Established Notation, Macmillan & Co., 1890, p. 1, Sec. 2. The two kinds of motion of the voice would then correspond to particular cases of musical and non-musical sounds - namely, the tones of the human voice when speaking and singing.

the human voice is a Fifth:

Dion. Hal. De Comp. Verb. XI:

διαλέκτου μὲν οὖν μέλος ἑνὶ μετρεῖται διαστήματι τῷ λεγομένῳ διὰ πέντε, ὡς ἔγγιστα καὶ οὔτε ἐπιτείνεται πέρα τῶν τριῶν τόνων καὶ ἡμιτονίου ἐπὶ τὸ ὀξύ, οὔτε ἀνίεται τοῦ χωρίου τούτου πλεῖον ἐπὶ τὸ βαρύ.

Of course the manner and scope of this variation will be different in different languages. For German, Prof. Helmholtz gives a range of an octave, affirmative sentences ending with a fall of a Fourth, and interrogative sentences with a rise of a Fifth, from the mean pitch.[1] In some languages emphasis is indicated by a rise in pitch; in others, as Swedish, by a fall. (See Helmholtz - Ellis - p. 23.)

The passage in Aristoxenus where the two kinds of motion are identified with the singing and speaking voice runs as follows:

p. 9, l. 20, Ed. Meibom.

τὴν μὲν οὖν συνεχῆ, λογικὴν εἶναί φαμεν

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1. Sensations of Tone, 2nd English Ed. 1885, by Alexander J. Ellis, p. 238. As is done in this work, the terms, Octave, Fifth, Tone, etc., when used as the names of intervals, are here written with a capital initial letter to prevent ambiguity

διαλεγομένων γὰρ ἡμῶν, οὕτως ἡ φωνὴ
κινεῖται κατὰ τόπον, ὥστε μηδαμοῦ
δοκεῖν ἵστασθαι. κατά γε τὴν ἑτέραν,
ἣν ὀνομάζομεν διαστηματικήν, ἐναντί-
ως πέφυκε γίγνεσθαι. ἀλλὰ γὰρ ἵστασθαί
τε δοκεῖ, καὶ πάντες τὸν τοῦτο φαινό-
μενον ποιεῖν, οὐκέτι λέγειν φασίν,
ἀλλ' ᾄδειν. διόπερ ἐν τῷ διαλέγεσθαι
φεύγομεν τὸ ἱστάναι τὴν φωνήν,
ἂν μὴ διὰ πάθος ποτὲ εἰς τοιαύτην
κίνησιν ἀναγκασθῶμεν ἐλθεῖν·
ἐν δὲ τῷ μελῳδεῖν τοὐναντίον ποι-
οῦμεν. τὸ μὲν γὰρ συνεχὲς φεύγομεν·
τὸ δὲ ἑστάναι τὴν φωνὴν ὡς μάλιστα
διώκομεν, ὅσῳ γὰρ μᾶλλον ἑκάστην
τῶν φωνῶν μίαν τε καὶ ἑστηκυῖαν,
καὶ τὴν αὐτὴν ποιήσομεν, τοσούτῳ
φαίνεται τῇ αἰσθήσει τὸ μέλος ἀκρι-
βέστερον. ὅτι μὲν οὖν δύο κινήσεων
οὐσῶν κατὰ τόπον τῆς φωνῆς, ἡ μὲν συν-
εχὴς λογική τίς ἐστιν, ἡ δὲ διαστηματικὴ

μελῳδική, σχεδὸν δῆλον ἐκ τῶν εἰρημένων

We have seen that the distinction between the two sorts of motion of which the voice and a few instruments are capable, namely, the distinction between the continuous and the discontinuous motions, derives its importance from the fact that the melody which accompanies speech was felt to exist and was compared with the melody of formal music. It has also been shown, it is hoped, that when this difference between the two kinds of motion is analyzed, it turns out to be really more a difference of steadiness and unsteadiness in pitch than a difference of motion. In κίνησις συνεχής the pitch is nowhere steady, in κίνησις διαστηματική it is steady now at this height, now at that. If now we unite these two states of pitch, and imagine a sound, first to resound steadily at any pitch, then to move so that the pitch is sharpened or flattened continuously, and then again to resound steadily, we have the phenomenon called <u>portamento</u>. It resembles κίνησις συνεχής in having the same sort of motion, but has in common with κίνησις διαστηματική the characteristic steadiness of pitch on which alone can a true music be based and without which it would be impossible even to gain an idea of a musical inter-

val. As to its place in a scheme in which the kind of motion was considered, it would have to be classed under continuous motion, modified by periods of rests. Aristoxenus did not overlook this form of pitch-movement but recognized the fact of its existence, while dismissing it from consideration. Aristoxenus, Harmonic Elements, p. 9, Ed. Meib. Sec. 27.

ληπτέον δ' ἑκάτερον τούτων [i.e. the two forms of motion] κατὰ τὴν αἰσθήσεως φαντασίαν. πότερον μὲν γὰρ δυνατὸν ἢ ἀδύνατον φωνὴν κινεῖσθαι καὶ [ἢ Meib.] πάλιν ἵστασθαι ἐπὶ μιᾶς τάσεως ἑτέρας ἐστὶ σκέψεως, καὶ πρὸς τὴν ἐνεστῶσαν πραγματείαν οὐκ ἀναγκαῖον, τὸ δὲ κινηθῆναι τούτων ἑκάτερον. ἀμφοτέρως γὰρ ἔχει τὸ αὐτὸ ποιεῖν πρός γε τὸ χωρίσαι τὴν ἐμμελῆ κίνησιν τῆς φωνῆς ἀπὸ τῶν ἄλλων κινήσεων

But it is not easy to see why the whole of continuous motion should not be omitted, if any part of it is. Music proper begins when fixed degrees of pitch are selected for use in the construction of tunes. "The first fact that

we meet with in the music of all nations, so far as is yet known, is that alterations of pitch in melodies take place by intervals, and not by continuous transitions." (Helmholtz-Ellis, p. 250.) But we must take the Greek treatment of the subject as we find it.

From the latter half of the passage quoted at p. 27, l.12 ( ἐν δὲ τῷ μελῳδεῖν, κτέ. ) it would seem that portamento was avoided as much as possible in singing.

Aristides Quintilianus makes a three-fold division of κίνησις, in which the portamento is provided for.

Arist. Quint. De Mus. p.7, Ed.Meib:

ἡ δὲ ἁπλῆ κίνησις συνέστηκεν ἐν διαφόροις χρόνοις. χρόνος μὲν ὅσον μέτρον κινήσεως, καὶ στάσεως, τῆς δὲ κινήσεως ἡ μὲν ἁπλῆ τέτυχεν, ἡ δὲ οὐχ ἁπλῆ· καὶ ταύτης ἡ μὲν συνεχής, ἡ δὲ διαστηματική, ἡ δὲ μέση. συνεχὴς μὲν οὖν ἐστι φωνή, ἡ τάς τε ἀνέσεις καὶ τὰς ἐπιτάσεις λεληθότως διά τε τάχος ποιουμένη. διαστηματικὴ δὲ ἡ τὰς μὲν τάσεις φανερὰς ἔχουσα, τὰ δὲ τούτων μέτρα λεληθότα. μέση δὲ ἡ ἐξ ἀμφοῖν συγκειμένη· ἡ μὲν οὖν συνεχής ἐστι, ᾗ διαλεγόμεθα. μέση δέ, ᾗ τὰς τῶν ποιημάτων ἀναγνώσεις ποιούμεθα. διαστηματικὴ δέ, ἡ καὶ τὰ μέσον τῶν ἁπλῶν φωνῶν ποσὰ ποιουμένη διαστήματα, καὶ μονὰς ἥτις καὶ μελῳδικὴ καλεῖται.

We must now consider the method employed by the great Pythagorean (using the term in its musical application), Caludius[1] Ptolemy, the Alexandrian mathematician, astronomer,

and geographer. His Harmonics in three books is of equal importance with the musical works of Aristoxenus and Aristides Quintilianus. Ptolemy may be considered as the representative of the more mathematical of the two great rival schools in musical theory - the Aristoxeneans and the Pythagoreans. Aristides is classed by Gevaert as an eclectic.

In order to fix the position of musical sounds in relation to other sounds Ptolemy proceeds in his treatise in the following manner:

After two preliminary chapters, we come to a chapter on the nature of pitch, and then to one on sounds and their differences. The word for sound in general is ψόφος, which Wallis renders by <u>sonitus</u> in his Latin translation. The chapter is entitled Περὶ φθόγγων, καὶ τῶν ἐν αὐτοῖς διαφορῶν but φθόγγων must be wrong, for φθόγγος is technically restricted to the meaning 'musical sound'. The separation of sounds into two classes according as pitch is absent or present is implied, unless to be sure pitch is predicated of all sounds whatsoever. The first classification we meet with is that of sounds which have pitch into two

groups according to the nature of that pitch.

ψόφοι, he says ( Harmonics I.iv.8,Ed. Wallis ) are either ἰσότονοι or ἀνισότονοι; ψόφοι ἰσότονοι are sounds which are unchangeable in the matter of pitch; ψόφοι ἀνισότονοι are those which change their pitch. His words are:

τούτων τοίνυν οὕτως ἐχόντων διοριστέον ἐφεξῆς ὅτι τῶν ψόφων οἱ μέν εἰσιν ἰσότονοι, οἱ δ' ἀνισότονοι. ἰσότονοι μὲν οἱ ἀπαράλλακτοι κατὰ τὸν τόνον, ἀνισότονοι δὲ οἱ παραλλάσσοντες.

The word here rendered 'pitch', τόνος, Ptolemy himself hastens to define in the following sentence, quoted at p.19: ὁ γὰρ αὐτὸς λεγόμενος τόνος κοινὸν ἂν εἴη γένος τῆς ὀξύτητος καὶ τῆς βαρύτητος, παρ' ἓν εἶδος τὸ τῆς τάσεως εἰλημμένος, ὡς τὸ πέρας τοῦ τέλους καὶ τῆς ἀρχῆς.

This equality and inequality of pitch refers of course to the possibility of change which any sound may undergo in the course of its existence. But the terms, ἰσότονος and ἀνισότονος, are also used in a very different sense, a sense which is met with in the very next chapter of Ptolemy's treatise, and indeed even at the end of the present chapter. Ambiguity in the use of terms has always been in music a peculiarly fertile cause of misunderstanding. The difference in the meanings is well stated by Porphyry in his comment on this passage (Commentarius Cap. iv. p. 258, Ed. Wallis). The other use of the words will not cause difficulty, as the difference is so clear. In this meaning the τόνος refers to the pitch of notes as compared with that of other notes, and used in this way the terms are of frequent service in demonstrations of the Pythagorian theory of consonant and dissonant intervals. Porphyry says p. 258, Ed. Wallis:

Ῥητέον πάλιν ὡς ἰσότονος ὁ φθόγγος λέγεται διχῶς· ὁ μὲν ἄλλῳ φθόγγῳ ἴσην τὴν τάσιν κεκτημένος, ὥσπερ ἡ νήτη συνημμένων τῇ παρανήτῃ διεζευγμένων λέγεται εἶναι ἰσότονος.

(We might say that E# in the key of B major was 'equitonic' with F.) People call such an ἰσότονος ψόφος more properly ὁμότονος ( and not merely ψόφος but φθόγγος ). The other meaning, Porphyry continues, refers to parts of one and the same sound, (p. 259,) as the beginning, middle, and end. Such a sound might with more exactness be called ὁμοιομερής.

Ptolemy next takes ψόφοι ἀνισότονοι and divides them into οἱ συνεχεῖς and οἱ διωρισμένοι. Definitions follow.

"Continuous sounds ( ψόφοι συνεχεῖς ) are those, the regions of whose changes ( in pitch) in each direction are not manifest, or of which no part whatever is ἰσότονος for a perceptible interval ( of time). The same sort of thing is seen in the case of the colors of the rainbow. Of such a nature are those sounds which sound at the same time as their pitch is being raised or lowered, even while this change is being produced." The lowing of cattle and howling of wolves are given as examples of these continuous sounds. Porphyry adds the attempts of beginners at singing, who cannot strike the right pitch at first, but feel for it ( ζητεῖ ἅψασθαι τὸν σύμφωνον τόπον

τῆς ἐκτεθείσης τάσεως· καὶ ὀξυτέρας μὲν τῆς ἰδίας προφορᾶς αἴσθησιν λαβών, παροξύνει κατ' ὀλίγον, ὥστε τὸν διάστημα μηδὲν ποιῶν· ὀξύνας δέ, βαρύνει πάλιν πρὸς ὀλίγον· ταῦτα δὲ ποιῶν συνεχῆ μὲν τὴν τάσιν τῆς φωνῆς ποιεῖ, ἐπὶ μίαν δὲ καὶ ὁμοίαν καὶ ἴσην τάσιν οὐκέτι, οὐδὲ ἰσοτόνως. (p. 260, Ed. Wallis.) He also gives as an instance the tuning of stringed instruments. All such continuous sounds are unfit for music (harmonic).

Ptolemy's second class, ψόφοι (ἀνισότονοι) διωρισμένοι - 'discrete' sounds - he defines as those sounds, "the region of whose changes are manifest," that is, the spaces or lengths - pitch distances - traversed are measurable; or such sounds arise when their parts remain 'equitonic' for a perceptible interval. As a parallel he gives the juxtaposition of pure and unmixed colours. Sounds like these are suitable for music (harmonic), because they are bounded by ἰσότονοι ψόφοι and may be compared by the order of their excesses ( ὑπεροχαί ) that is, by their ratios.

Continuous sounds are, then, sounds in which the variation in pitch takes place in such a way that it advances by insensible gradations and that it has no definite limits. Discrete sounds are those in which the pitch moves through well defined distances. The two classes are characterized precisely by the two kinds of κίνησις explained by Aristoxenus - the continuous and the intervallate ( κίνησις συνεχής and κ. διαστηματική ).

Like ~~the~~ Aristoxenus' classification of pitch motion the Ptolemaic classification of sounds is not quite rigorously logical. The species do not together cover as much ground as the genus of which they are species. As in the other case, there is no place for slurred sounds, which must be classed under φθόγγοι ἀνισότονοι , or, at any rate the slur, or rather the portamento itself, must be so classed. But these sounds cannot come under the definition of φθόγγοι συνεχεῖς , because the latter have no definite bounds to the pitch movement; on the other hand they differ from διωρισμένοι φθόγγοι in that the pitch advances by glides and not by steps.

Furthermore φθόγγοι διωρισμένοι are by

definition taken out of the class of ψόφοι ἀνισότονοι to which they profess to belong, because they may be analyzed into groups or series of ψόφοι ἰσότονοι

The whole difficulty ( and it is one which must be guarded against in every classification of sounds) is that what is understood by a single sound is not an instantaneous sensation of the auditory nerves, which we may consider apart from its surroundings, as if cut off from both what sounded before and what is to sound afterward. Nor, on the other hand, are we justified in regarding too long a duration of tone as forming a unit. Φθόγγοι διωρισμένοι are really ψόφοι ἰσότονοι . The classification must have been traditional from the time of Aristoxenus, if not earlier.

We have noticed ( page 35 ) that Ptolemy regarded ψόφοι διωρισμένοι as suitable for the science of music, but ψόφοι συνεχεῖς as unsuitable. The latter have no unity ( μηδέχνη μηθὲν ὑποβάλ-λοντες ἓν καὶ ταὐτό. Ptol. Harm.I.,iv,Ed.Wallis, p. 8, last line.) The former Ptolemy, after the definitions are completed, calls φθόγγοι

Ptol. Harm. p. 9 beginning, Ed. Wal. καὶ δὴ φθόγγους ἤδη καλοῦμεν ἐν τοῖς ἰσοτόνους· ὅτι φθόγγος ἐστὶ ψόφος ἕνα καὶ τὸν αὐτὸν ἐπέχων τόνον

Ψόφοι ἰσότονοι are not included by name; but they are plainly embraced by the definition. It is possible, however, that in the primary division ( into ψόφοι ἰσότονοι and ψ. ἀνισότονοι ) Ptolemy had in mind masses of tone or large groups of sound, and that, as an example of ἰσότονοι ψόφοι he might have given the tones of an instrument like the whistle or the cymbals or the horn or any instrument which can produce only one note. The term φθόγγος would in this case be restricted to musical notes which occurred in melody.

The definition of the musical φθόγγος varies in the Greek treatises on theory according to the manner in which the subject is approached.

Aristoxenus, as we have seen, makes the development of the subject rest on an analysis of the κίνησις φωνῆς We are not then surprised to find that φθόγγοι are the elements of κίνησις διαστηματική. We find it said at p. 12 of the voice κινεῖσθαι μὲν γὰρ ἐν τῷ διαστήματι ποιεῖν,

We may tabulate the definitions as follows:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Aristoxenus - - - | | φωνῆς | – | πτῶσις | ἐπὶ | μίαν | τάσιν |
| Gaudentius - - - | | " | | " | " | " | " |
| Bacchius - - - | | " | ἐμμελοῦς | " | " | " | " |
| Aristoxeneans (Porphyry) - - - | | " | " | " | " κατὰ | " | " |
| " | ⎱ ἐμμελὴς | " | — | " | ἐπὶ | " | " |
| Bryennius - | ⎰ — | " | – | " | ἐμμελὴς ἐπὶ μίαν | | " |
| Nicomachus - - | | " | " | ἀπλατὴς | | | τάσις |
| Thrasyllus - - | | " | ἐναρμονίου | – | | | " |
| Bryennius - - | | " | ἐναρμόνιος | | | | " |
| " - - | | φωνὴ | ἀσχηματική | | | | |
| " - - | | μιᾶς χορδῆς | ~~μέρος ἐλάχιστον~~ τοιχ[illegible]ῖς, ἀπήχησις | | | | |
| Arist. Q. - - | | φωνῆς | ἐμμελοῦς | μέρος ἐλάχιστον | | | |
| Porphyry | | ἕν τι μέρος ἐλάχιστον τοῦ μέλους | | | | | |
| Ptolemy | | ψόφος ἕνα καὶ τὸν αὐτὸν ἐπέχων τόνον | | | | | |
| Pythagoreans (acc. to Porphyry) | | " | ἠχὴ μίαν τάσιν ἐκφερόμενος | | | | |

ἵσταται δ' ἐν τῷ φθόγγῳ.

The stopping places of κίνησις διαστηματική are φθόγγοι. At p. 15 the definition is slightly different: φωνῆς πτῶσις ἐπὶ μίαν τάσιν, ὁ φθόγγος.

The definition of Thrasyllus we owe to Theo Smyrnaeus.

Theo Sm. De Musica, c. 2, p. 47, fol. Ed. Hiller: Θράσυλλος τοίνυν περὶ τῆς ἐν ὀργάνοις αἰσθητῆς λέγων ἁρμονίας φθόγγον φησὶν εἶναι φωνῆς ἐναρμονίου τάσιν. ἐναρμόνιος δὲ λέγεται, ἐπεὶ δύναται καὶ τοῦ ὀξέος ὀξύτερος εὑρεθῆναι καὶ τοῦ βαρέος βαρύτερος· καὶ ὁ μέσος αὐτὸς καὶ μέσος ἐστίν. ὡς εἴ γέ τινα τοιαύτην φωνὴν νοήσαιμεν ἥτις ὑπερέχει πᾶσαν ὀξύτητα, οὐκ ἂν εἴη ἐναρμόνιος· οὐδὲ γὰρ τὸν τῆς ὑπερμεγέθους βροντῆς ψόφον ἐναρμόνιον ἔχομεν, ὅς γε καὶ ὀλέθριος διὰ τὴν ὑπερβολὴν πολλάκις γίνεται, ὥς τις ἔφη·

πολλοὺς δὲ βροντῆς ἠχὼ ἔκτεινεν ὤλεσε.

καὶ μὴν εἴ τις οὕτως βαρὺς εἴη φθόγγος, ὡς

μὴ ἔχειν αὐτοῦ θατέρου, οὐκ ἂν οὐδὲ φθόγγος εἴη τὸ ἐμμελὲς οὐκ ἔχων. διὰ τοῦτ' οὖν φθόγγος εἶναι λέγεται οὐ πᾶσα φωνὴ οὐδὲ πάσης φωνῆς τάσις, ἀλλ' ἡ ἐναρμόνιος οἷον μέσης, νεάτης, ὑπάτης.

Quoted verbatum, (except ll. 12, 13 ὅς γε ... ἄλλοις )

by Bryennius p.

The restriction imposed by the word ἐναρμόνιος confines the musical sound to the limits of the recognized scale.

Ptolemy defines φθόγγος as follows:

Harmonics, I. c. iv. fin. Wallis, p. 9:

φθόγγος ἐστὶ ψόφος ἕνα καὶ τὸν αὐτὸν ἐπέχων τόνον.

Nichomachus Harmonics Manuale, Meib. p. 7:

καθόλου γάρ φασιν ψόφον μὲν εἶναι πλῆξιν ἀέρος ἄθρυπτον μέχρι ἀκοῆς· φθόγγον δέ, φωνῆς ἐμμελοῦς ἀπλατῆ τάσιν· τάσιν δέ, μονήν τινα καὶ ταυτότητα κατὰ μέγεθος φθόγγου ἀδιάστατον

Porphyry in his commentary on the passage of Ptolemy quoted above says that Ptolemy changed the usual definitions of φθόγγος.

Porphyry Comm. c. iv., p. 262: Εἶτα ἀποδίδωσιν ὅρον τοῦ φθόγγου, φθόγγος γάρ ἐστι ψόφος ἕνα καὶ τὸν αὐτὸν ἐπέχων τόνον, τόνον μὲν λαμβάνων ἀντὶ τῆς τάσεως καθάπερ ἤδη κέχρηκε τοὺς δὲ φερομένους ὅρους τοῦ φθόγγου μεταλλάξων. φέρονται γὰρ αὐτοῦ ὅροι παρὰ μὲν τοῖς Πυθαγορείοις, φθόγγος ἐστὶ ψόφος παρὰ μίαν τάσιν ἐκφερόμενος, παρὰ δὲ τοῖς Ἀριστοξενείοις φθόγγος ἐστὶ φωνῆς ἐμμελοῦς

ἢ ἐμμελὴς φωνῆς

πτῶσις ἐπὶ μίαν τάσιν, φωνῆς μὲν ἐμμε-

πτῶσις ἐπὶ μίαν τάσιν.

λοῦς εἴρηται ἐπείπερ οὐ περὶ πάσης φωνῆς ὁ λόγος ἀλλὰ τινος, τουτέστι τῆς ἐμμελοῦς, ἐμμελῆ δὲ φωνὴν τὴν αὐτὴν τῇ διαστηματικῇ τιθέμενος. Ὅθεν δυνάμει τὸ λεγόμενον μέν ἐστι φωνῆς διαστηματικῆς. διαστηματικὴ δὲ φωνή ἐστιν ἡ πρὸς μέλος ἐπιτήδειος, ἣν διαστέλλουσιν πρὸς τὴν κατὰ τὰς ὁμιλίας εἰς τὴν χρῆσιν παραλαμβανομένην ἣν συνεχῆ καὶ λογικὴν καλεῖν εἴωθεν ὁ Ἀριστόξενος· πτῶσις

τὸ δοκεῖν τὴν μὲν συνεχῆ ὥσπερ ἄλογον εἶναι, τὴν μέντοι διαστηματικὴν τὴν ἐναντία μὴ ἀξιοῦσι καλεῖσθαι. καὶ μονοειδῆ ἀπὸ τοῦ ἑστάναι πεπονθυῖαν ἐμμελῆ γεγονέναι. διὸ καὶ τὸ μέλος ἀποδίδωσι, κλάσιν φωνῆς. — — — τὸ δὲ ἐπὶ μίαν τάσιν, ἐπεὶ τὸ μὲν ὅλον μέλος πτῶσις ἐστί, καὶ πολλὰς τάσεις, καὶ τοσαύτας ὅσας ἐκεῖνα παρέχει κατὰ τὸ σύστημα. ὁ δὲ φθόγγος ἕν τι μέρος ἐλάχιστον ὢν τοῦ μέλους, ἐξ ἀνάγκης, καὶ τὴν ἐφ᾽ ἑαυτὸν γενομένην πτῶσιν μίαν ἔχει.

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ἐπεὶ οὖν παρὰ τοῖς Ἀριστοξενείοις ἀπεδόθη, τὸ εἶναι τὸν φθόγγον φωνῆς ἐμμελοῦς πτῶσιν καὶ μίαν τάσιν ἐκφερομένην τοῦτο μετείληπται εἰς τὸ εἶναι τὸν φθόγγον, ψόφον, ἀλλὰ καὶ τὸν αὐτὸν ἀπέχοντα τόνον.

Aristides Quintilianus, De Musica, p. 9,l. 2, Ed. Meib:

πᾶσα μὲν οὖν ἁπλῆ κίνησις φωνῆς, τάσις· ἡ δὲ τῆς μελῳδικῆς, φθόγγος ἰδίως καλεῖται.

p. 9. l. 17 φθόγγος μὲν οὖν ἐστι, φωνῆς ἐμμελοῦς μέρος ἐλάχιστον.

Gaudentius defines φθόγγος in the same words as Aristoxenus: φωνῆς πτῶσις καὶ μίαν τάσιν, and adds Aristoxenus's definition (at Meib.p.12) of τάσις The passage reads as follows:

Harm. Introd. p. 2 φθόγγος δέ ἐστι φωνῆς πτῶσις ἐπὶ μίαν τάσιν. τάσις, δέ, μονὴ καὶ στάσις τῆς φωνῆς. ὅταν οὖν ἡ φωνὴ κατὰ μίαν ~~φω-~~ ~~νῆς~~ τάσιν ἑστάναι δόξῃ, τότε φαμὲν φθόγγον εἶναι τὴν φωνὴν οἷον εἰς μέλος τάττεσθαι.

Bacchius Senior in his Introductio Artis Musicae modifies the definition of Aristoxenus by adding ἐμμελής, as Porphyry does.

p. 1 and 2: φθόγγος δὲ καθόλου τί ἐστι; φωνῆς ἐμμελοῦς πτῶσις ἐπὶ μίαν τάσιν. μία γὰρ τάσις ἐν φωνῇ ληφθεῖσα ἐμμελῆ φθόγγον ἀποτελεῖ.

Bryennius, Harmonics Bk. I. Sect. iv., (Ed. Wallis) p. 377, l. 9: φθόγγος ἐστὶ φωνῆς πτῶσις ἐμμελὴς ἐπὶ μίαν τάσιν, ἤτοι φωνὴ σχηματική and l. 17 ἢ φθόγγος ἐστὶ φωνῆς ἐνάρμονιος τάσις.

and l. 29 ἢ σαφέστερον εἰπεῖν φθόγγος ἐστὶ μιᾶς χορδῆς ποιά τις ἀπήχησις.

Two notes, or musical sounds, are said to form an interval when they differ in pitch; or, an interval is the difference of pitch between two notes.

The definitions of interval ( διάστημα ) in the Greek musical works will now be quoted for the most part in the order of time.

Aristoxenus (Harmonic p. 15, Ed.Meib)

διάστημα δέ ἐστι τὸ ὑπὸ δύο φθόγγων ὡρισμένον μὴ τὴν αὐτὴν τάσιν ἐχόντων

Thrasyllus's definition of διάστημα is found in Theo Smyrnaeus, De Musica, c. 3, p. 48, Ed. Hiller:

διάστημα δέ φησιν εἶναι φθόγγων τὴν πρὸς ἀλλήλους ποιὰν σχέσιν, οἷον διὰ τεσσάρων, διὰ πέντε, διὰ πασῶν.

Plutarch, De Animae Procreatione, XVII, 1020 E:

ἔστι γὰρ διάστημα ἐν μελῳδίᾳ πᾶν τὸ περιεχόμενον ὑπὸ δυοῖν φθόγγων ἀνομοίων τῇ τάσει.

Ptolemy's Harmonics do not seem to contain a definition of διάστημα unless it is implied in the words ἡ κατὰ τὸ ὀξὺ καὶ βαρὺ τῶν ψόφων διαφορά.

Ptol. Harm. I. iii., p. 7, Ed. Wallis:

ἔοικεν ἡ κατὰ τὸ ὀξὺ καὶ βαρὺ τῶν ψόφων διαφορὰ ποσότητος εἶδος εἶναι.

Aelian, as quoted by Porphyry, Commentarius in Ptolemaeum, Ed. Wal. p. 217 fin.

Συμφωνὰς ἐστι ὀξὺς φθόγγος ὑπὸ

βαρυτέρου διάστημα ἀφεστήκασι· καὶ ἡ διαφορὰ τοῦ ὀξυτέρου παρὰ τὸν βαρύτερον φθόγγον, καὶ τοῦ βαρυτέρου παρὰ τὸν ὀξύτερον καλεῖται διάστημα.

p. 218 καὶ οὕτως ὁρίζεται τὸ διάστημα· ὅ ἐστι φθόγγων ἀνομοίων ὀξύτητι καὶ βαρύτητι διαφέρον.

Nicomachus, Harmonices Manuale Meib. p. 7.

Διάστημα δὲ τὴν ὁδὸν ποιὰν ἀπὸ βαρύτητος εἰς ὀξύτητα ἢ ἀνάπαλιν.

Bacchius Senior, Introductio p. 2:

διάστημα τί ἐστι; διαφορὰ δύο φθόγγων ἀνομοίων ὀξύτητι καὶ βαρύτητι.

Gaudentius, Harmon. Introd. p. 4:

διάστημα δέ ἐστι τὸ ὑπὸ δύο φθόγγων περιεχόμενον

Bryennius, Sectio V., p. 381, (Wal):

τὸ διάστημα τοίνυν διχῶς λέγεται κοινῶς καὶ ἰδίως. καὶ κοινῶς μὲν καὶ μέγεθος τὸ ὑπὸ τινῶν περάτων ὁριζόμενον.

ἰδίως δὲ κατὰ τὴν μουσικὴν οὕτως ὁρίζεται, διάστημά ἐστι μέγεθος φωνῆς ὑπὸ δυοῖν φθόγγων περιγεγραμμένον· ἤ, διάστημά ἐστι τὸ περιεχόμενον ὑπὸ δύο φθόγγων ἀνομοίων τῇ τάσει, ἤτοι ὀξύτητι καὶ βαρύτητι· ἢ ὁδὸς ποιὰ ἀπὸ βαρύτητος εἰς ὀξύτητα ἢ ἀνάπαλιν· ἢ διάστημά ἐστι, τὸ ὑπὸ δύο φθόγγων ὡρισμένον, μὴ τὴν αὐτὴν ἐχόντων τάσιν· ἤτοι διαφορά τις τάσεων καὶ τόπος δεκτικὸς φθόγγων ὀξυτέρων μὲν τῆς βαρυτέρας τῶν ὁριζοντῶν τὸ διάστημα τάσεων, βαρυτέρων δὲ τῆς ὀξυτέρας. διαφορὰ δέ ἐστι τάσεων, τὸ μᾶλλον καὶ ἧττον τετάσθαι. ἢ διάστημά ἐστι, δυοῖν φθόγγων ἡ πρὸς ἀλλήλους ποιὰ σχέσις.

It will be noticed that Bryennius has collected all previous definitions and presents them as alternatives.

For the sake of bringing out more clearly the differences in these definitions, we may group them in classes. Intervals are defined in one of the following manners:

(1) Interval is a certain <u>relation</u> between two <u>sounds</u>-

musical sounds. (Thrasyllus, Bryennius.)

(2) Interval is a difference of pitches, i. e., of tension (Aristoxenus, Bryennius), or the difference between an acute sound and a grave sound, or between two sounds not of the same pitch. (Ptolemy, Aelian, Bacchius.)

(3) Interval is that which is bounded or contained by two sounds not of the same pitch, (Aristoxenus, Plutarch, Gaudentius, Bryennius,) or a region or space ( τόπος ) receptive of sounds intermediate in pitch to the extremes , (Aristoxenus, Bryennius,) or a vocal magnitude ( μέγεθος ) defined or bounded by two sounds. (Bryennius.)

(4) Interval is a passage or passing ( ὁδός ) from acute to grave or vice versâ. (Nicomachus, Bryennius.)

Dismissing (1) because it is too indefinite, we may notice that the definitions numbered (2) merely express the fact that it is pitch-difference on which the relationship depends, but do not imply that there is such a thing as difference in the size of intervals and that measurement is possible. The difference of pitches might be like the difference of weights, as the difference between two bodies weighing two and three pounds, or a difference in two shapes. In the definitions numbered (3) the idea of a space or distance is clearly brought

out, and in (4) the notion of movement from one bound to the other is included. Both (3) and (4) imply more or less clearly the infinite subdivisibility of pitch. This is very plain in the definition that an interval is a space receptive of intermediate pitches, for an intermediate pitch would form two new smaller intervals, which would themselves also be divisible into yet smaller intervals.

In (4), and to a certain extent also in (3), κίνησις συνεχής seems to lie at the bottom of the conception of an interval. Without this idea of a gradual passage from one pitch to another, it is hard to see how change of pitch could be regarded as a passage at all. If the pitch changes suddenly, in the way indicated by κίνησις διαστηματική the sensation does not suggest a transition or transference, so much as a transformation. The effect is similar to that produced by a sudden change in color. Intermediate shades of color are not present in the mind. The continuous nature of pitch would however be one of the earliest points to be observed. It would readily be admitted that for practical purposes pitch seemed to be a continuous quantity and not a discrete quantity. When however the question of finding a means of measuring difference of pitch was presented, it

would be natural to endeavor to find a smallest possible interval which might serve as a natural unit.

We have in Plato's Republic Bk. VII, p. 531, A, fol. a reference to such attempts.

530 E: ἢ οὐκ οἶσθ' ὅτι καὶ περὶ ἁρμονίας ἕτερον τοιοῦτον ποιοῦσι; [531 A] τὰς γὰρ ἀκουομένας αὖ συμφωνίας καὶ φθόγγους ἀλλήλοις ἀναμετροῦντες ἀνήνυτα, ὥσπερ οἱ ἀστρονόμοι, πονοῦσι. Νὴ τοὺς θεούς, ἔφη, καὶ γελοίως γε, πυκνώματ' ἄττα ὀνομάζοντες καὶ παραβάλλοντες ὦτα, οἷον ἐκ γειτόνων φωνὴν θηρευόμενοι, οἱ μέν φασιν ἔτι κατακούειν ἐν μέσῳ τινὰ ἠχὴν καὶ σμικρότατον εἶναι τοῦτο διάστημα, ᾧ μετρητέον, οἱ δὲ ἀμφισβητοῦντες ὡς ὅμοιον ἤδη φθεγγομένων, ἀμφότεροι ὦτα τοῦ νοῦ προστησάμενοι.

(Quoted in Theo Smyr. p. 6, Hiller.)

But it is evident in dealing with the sensation of pitch, that there is the widest range for differences of individual opinion, and even if a considerable number of competent persons could agree that some given interval was the smallest differ-

ence of pitch which they could distinguish from unison, there would still exist the necessity for finding a method of recording the width of this interval and this record must necessarily rest on physical considerations and so final appeal would again lie to the intellect. Pitch differences, regarded as sensation, are restricted on the side of close approximations or of minute intervals by limitations of a physiological nature - limitations which differ widely with the individual. Pitch, regarded as a conception and not as a perception, is capable of infinitely minute gradations. There can be no interval so small that we cannot conceive of a tone intermediate to the extremes of the intervals. "The pitch of a note depends upon the period of its vibration" that is, upon the time taken to complete one vibration. It is only necessary then to suppose a note whose period is intermediate in point of duration to those of the given notes.

The ancients recognized the theoretically perfect divisibility of pitch. The subject was treated by them under the head of interval. They asked the questions, What is the smallest possible interval? and, What the greatest?

Aristoxenus discussed the question at p. 13, fin. Meib.

Intervals are of different sizes. Of course this size

must be independent of the positions of the bounding notes on the scale of acuteness and graveness. What is meant by the size of an interval formed by two notes ( not at the same pitch), is a certain relation between their pitches, which may be called the difference in the pitches, the word 'difference' being used merely as the opposite of 'sameness'. This relation is recognized as being the same in all essential features in every part of the audible compass of sounds, however much the character of the interval may vary with the absolute pitch of the combination. It is well known that even consonances, if sounded in the lower register, take on a decidedly harsher character, if they do not actually become dissonant in the accepted meaning of that term. But the <u>size</u> of the interval is as certainly recognized at one pitch as at another.

For the measurement of the size of intervals, the first requisite would seem to be a unit of measurement. It is a characteristic peculiar to Music, that, unlike the other arts, it makes only the most sparing use of the material at its command, so far as pitch is concerned. It is said that within the compass of an average voice two or three hundred different degrees of pitch are distinguishable. If we suppose the compass of such a

voice to be an Octave and a Fifth, we find only twenty of these different pitches represented on keyed instruments, and although this number is considerably increased in practice, when temperament is ignored, it is still evident that there is a great disparity between the number capable of being produced and the number actually employed. Many, to be sure, compute that to represent accurately on a keyed instrument all the requisite notes, it would be necessary to have as many as twenty-five keys or even a larger number, within each octave, but many of these it is impossible to bring together in one piece of music. They belong to unrelated keys and are often so close as to be indistinguishable in pitch. It is equally true of Greek music that, although a large number of notes within each octave was recognized, (twenty-five, constituting 24 quarter tones) they were not all of them usable in the same piece of music. Aristoxenus is our authority for the statement that the voice cannot advance by quarter tones beyond the second step. The number of notes used at one time within an octave was never many more than eight.

An answer must now be given to the question, Is it possible to select for the unit one of the intervals found in actual use in music?

If we represent pitch by a straight line and suppose equal lengths on the line to represent equal intervals, wherever taken, and then arbitrarily choose a certain distance to represent any given interval, such as the Octave, it will of course be possible to find distances which will accurately represent all other intervals. The sum of any two such distances will then of course represent the sum of the two corresponding intervals obtained by making the acute note of one of them coincide with the grave note of the other. In like manner the arithmetical difference of two distances will represent the difference of two intervals, obtained by making the two acute notes or the two grave notes coincide.

But here we meet with a most characteristic property of musical distances. It is that not one of the intervals which the ear recognizes as having a special claim to a name and place in every musical system, intervals such as the Octave, Fifth, Fourth, Major Tone, etc., will be found to be commensurable[1] with any other. No two distances representing intervals furnished by nature itself can be expressed in terms of

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1. Two numbers are commensurable when there is a third number which is contained an exact number of times in each.

a common unit. Let us turn to the examination of a few of the intervals which are in a peculiar sense musical. Neither the Fifth nor the Fourth is an aliquot part of the Octave.

The Major whole Tone- the difference between the Fifth and Fourth - is an aliquot part of neither the Fifth nor the Fourth. Nor is the Tone an aliquot part of the Octave.

This fact, apart from all mathematical considerations, was recognized from very early times. The eleventh chapter of the first book of Ptolemy's Harmonics contains a demonstration that six Tones exceed an Octave and the amount of this excess, it is stated, is perceptible _even to the ear_. Earlier writers than Ptolemy prove mathematically that six Tones exceed an Octave by an interval, called the Pythagorean Comma, whose ratio is 524288:531441. Since this excess is slightly larger than the Comma of Didymus (80:81), which is the difference between the Major and the Minor whole Tone ( or between the true or just Major Third and the Pythagorean Major Third) Ptolemy's statement is not in the least incredible. All the difficulties sought to be obviated by the device of equal temperament arise from small intervals which are rarely larger than the Comma (80:81) by even a quarter of its size. If such intervals are felt by moderns, we cannot deny to the ancients ability to perceive small intervals of the same size. The existence of

quarter tones, at one period, at least, of the development of Greek music, points to a high degree of cultivation of the feeling for pitch differences among the ancients.

If then neither the Tone, the Fourth nor the Fifth are commensurable with the Octave, nor with one another, it only remains to be seen whether any small interval, used in Music or selected for this purpose from the multitude of unmelodious intervals, can be found which will serve as a measure of these and other musical intervals. The answer will be invariably the same. It is impossible, proceeding from an examination of all the intervals furnished by actual music, to find any small interval, musical or unmusical, which will enable us to express any given interval in terms of a common unit of measurement.[1] When this statement is made to apply to all intervals, however small, as well as to intervals like the Tone and Semitone, on whose suitability as accurate units even the ear can pass some sort of judgment, its truth must necessarily be proved by means of a mathematical demonstration. The mere statement of the fact will however suffice for the present. The cause for this state of affairs is found in the

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1. After selection of a unit has once been made it is of course possible to construct intervals which shall consist of

fact, now about to be touched on, that musical intervals, meaning thereby intervals which occur in actual music, are all expressible in mathematical ratios, which are derived from physical phenomena. As before noticed ( p. 49), pitch depends on the number of vibrations of the air generated in any fixed period of time by the cause of the sound. Consonance of pitches depends on the simplicity of the ratio between the vibration-numbers of the notes. And in general the ratio between the vibration numbers of any two notes belonging to the same system will have simple ratios, that is ratios which are inexpressible in terms of small numbers. Since, now, to compound two ratios it is necessary to multiply them together and not to add them, it follows that the sum of two intervals cannot be obtained by direct addition of their ratios, nor are ratios so related that a common constituent can be found which could serve as a measure of their relative size. "If we wish to have a measure of intervals in the proper sense, we

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two or more units, but this is evidently a reversal of the natural logical course of procedure, and the resulting intervals are generally not truly musical, but are approximations to real intervals.

must take, not the characteristic ratio itself, but the logarithm of that ratio - then, and then only, will the measure of a compound interval be the sum of the measures of the components," (The Theory of Sound, by Lord Raleigh, 1894, Vol. I., p. 7, Sec. 14 ) and when this has been done, all the logarithms will be incommensurable. But, of course, the size of any interval can be calculated to any required degree of accuracy.

This great fact of the incommensurability of musical intervals was known to the Greeks. That is to say, they were aware that the intervals used in music, so far as their size or width is concerned, cannot be connected by means of a common measure. This follows from the fact that they knew that intervals are expressible in terms of ratios. The Pythagorean school of musical theorists consistently denied that the Fifth and the Fourth were respectively equal to 3 1/2 and 2 1/2 Tones. But, as, in modern theory, these plain facts are consciously or unconsciously ignored, so in the ancient musical world we find the Aristoxenean school making the Semitone, defined as half of the whole Tone (which in turn is defined as the difference of the Fifth and the Fourth) serve as a unit for the measurement of all other intervals. It is in-

convenient, to say the least, to have no unit supplied by nature, and for the purposes of a practical notation, some sort of approximate unit would seem almost a necessity for a music developed to the point of demanding different keys. In teaching too, it is very desirable to be able to regard that scale, in which progression is made by the smallest steps recognized, as composed of equal-sized intervals.

For the rougher measurements, then, we are quite justified in assuming as a unit whatever interval we find most convenient for the purpose. The exact size of such an interval will of course depend on the nature of the music concerned. Thus in Hindu music the octave was regarded as consisting of 22 small intervals ( çrutis ), such that 9 of them make a Fourth, and 13 a Fifth, and consequently four went to the Major whole Tone.

In Arabian music 17 approximately equal intervals compose the octave.

Both ancient Greek music and modern European music divide the octave into twelve nominally equal small intervals called Semitones, of which five make up the Fourth and seven the Fifth, and two the Tone, which is their difference. But while in modern music no interval differing very widely from the

twelfth part of an Octave or from multiples thereof, is used, in ancient Greek music on the contrary, the existence at different periods of quarter and third-Tones equal to one-half and two-thirds of the Semitone respectively, seems to be well attested. For this reason perhaps the Tone and not the Semitone is the best unit for rough measurement. Whether or not the subtle refinements known as the Chroai, which were varieties of the quarter-tone system[1] and of the third-tone system,[2] corresponded to actual observation of facts, Greek theoretical writers used both thirtieths and and sixtieths of a Fourth in their explanations of these various genera. These intervals would then be twelfths and twenty-fourths of a compromise tone obtained by taking exactly two-fifths of a Fourth. It is easy to see that a mean Tone of this size is not equal to an equal-temperament Tone, because a Fourth falls short of five equal-temperament Semitones, and consequently, its fifth part falls short of one 'equal' Semitone, and two-fifths of an 'equal' Tone. Neither the thirtieth nor the sixtieth part of a Fourth is an aliquot part of the Octave. In their more accurate measurements of intervals, the Greeks used the Fourth as a

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1. Genus enharmonicum. 2. Genus chromaticum.

standard of length, where moderns use the Octave. In this and in other respects the Fourth played the part now taken by the Octave. For rougher calculations the Tone and the Semitone - the sixth and the twelfth of the octave respectively - were freely used. In one respect their music demanded a minuter subdivision of the unit of measurement than does modern music. In their enharmonic genus quarter tones were used. Although obsolete so early as the time of Aristoxenus, so far as practical music was concerned, they nevertheless continued to retain a place in theory, and even after their actual use had ceased, the notation bore indelible traces of their influence.

Opposed to the more practical school of musicians, of which Aristoxenus is the foremost representative, stood the Pythagorean school. Pythagoras is accredited with making the discovery of the numerical relations which exist between musical sounds, and the Pythagorean school make ratio the basis for the measurement of intervals. The Pythagoreans demonstrated that it is impossible to divide an interval into exact halves. (Of course exception is made of those intervals which are actually created by compounding two equal intervals.) This is virtually equivalent to proving the non-existence of a common

unit of measurement. But in ratio they found a perfect method of measuring intervals. When it was discovered that the wonderful superiority which a few intervals of certain definite widths possess over the unlimited number of comparatively characterless intervals of other sizes, depends on certain fixed arithmetical relations between the numbers which may be connected with the notes concerned, it must have been regarded as a signal confirmation of the Pythagorean doctrine of number. It is not impossible that this discovery was partly responsible for the origin of the doctrine. If the harmony of musical sounds, and the motions of the planets depend on harmony among numbers, it is a natural step to see the influence of number in all life.

Pitch is determined by the number of vibrations of the air made in a second or other given period of time. This number is called the vibration-number. The interval between any two notes depends on the ratio existing between these two vibration numbers. This ratio will, of course, remain the same for any and every period of time, and for all positions in the scale of pitch at which the interval may be found. Given any two notes, their interval may be calculated by finding their vibration-numbers and deducing the ratio. But the

ancients had no means either for counting the number of vibrations or for accurately measuring small intervals of time like the second. Consequently ancient determinations of the ratios of intervals were based on other considerations. The most convenient method consisted of a comparison of the lengths of the strings producing the required notes, when made of a uniform thickness and subjected to the same tension. As it happens the lengths of strings are inversely proportional to their vibration-numbers, so that results obtained by one method may easily be compared with those of the other when two notes only are involved, and without difficulty when there is a series. Other methods employed were the comparison of the lengths of the pipes of wind instruments of equal bore; the comparison of the distances at which finger holes must be bored to produce given notes; and the comparison of the weights necessary to stretch strings of equal length as well as size, so as to produce notes which will form the required interval. (cf. Theo Sm. p. 57, Hiller.) Only very rough results could have been obtained from these last methods. In the case of instruments like the flute, ( αὐλός ), it is very difficult to determine accurately the length of the vibrating column of air, and it is necessary that the bore of the instrument be

of uniform size throughout and that the size of the finger-holes be the same. A hole of smaller diameter may be substituted for one of larger diameter further removed from the mouth-piece. Ancient flute-makers undoubtedly availed themselves of this principle in tuning their flutes. For ascertaining interval ratios by measuring the distances at which the holes are placed, it would be necessary to have the holes of one size only. In using strings of equal length and thickness, stretched by hanging weights of different sizes, great care would have to be exercised. In order that two strings of equal length and size shall produce sounds which form some given interval, it is necessary to employ weights which are to each other inversely as the squares of the lengths of strings of equal size at the same tension producing the same intervals; or, the lengths vary inversely as the square-roots of the weights. The weights would not therefore give directly the proportions sought for. It is doubtful if the ancients could have obtained the musical ratios from weights. Allowance would also have to be made for the fact that the weight of the string per linear unit is diminished by the tension.

Ptolemy discusses the difficulties attending these methods in Bk. I., Chap. VIII, (p. 17, Wallis) of his Harmonics,

where he describes the instrument on which the greatest reliance was placed for determining the ratios - the κανὼν μονόχορδος ἁρμονικός. This instrument consisted of a string which passed over two fixed bridges and one movable bridge, which could pass from one end of the string to the other along a scale which ran beneath the string, and by means of which the distances between the movable bridge and the fixed bridges could be measured. In this way the ratios associated with the various musical intervals could be calculated. If the whole length of the string was tuned to be in unison with the lowest note of the scale of two octaves, called the Perfect System, the proper distances could be marked off for all the other notes. This operation was called ἡ τοῦ κανόνος κατατομή. We have a description of the method in which the string was divided in Theo Smyrnaeus, Expositio Rerum Mathematicarum ad Legendum Platonem Utilium, pp. 57-58, Ed. Hiller (De Mus. c. 12;) and pp. 87-93 (De Mus. cc. 35-36.) where Thrasyllus is quoted <u>in extenso</u>. We have also Euclid's treatise Sectio Canonis. See too Boeckh, De Metris Pindari, lib. III. c. vii. (Pindari Opera tom. I. pp. 209, 210); Kleine Schriften III Ueber die Bildung der Weltseele im Timaeos der Platon p. 66 (p. 150) An important advantage giv-

en by the Monochord was that the tension of the two parts of the string is necessarily the same. The element of tension is thus eliminated, and, provided care is taken to make the string of uniform thickness and weight and to have sharply defined termini, the lengths of the two parts of the strings, or of a part to the whole, may be directly compared.

The question now arises, What are the ratios which must exist between two lengths of string in order to produce given intervals? As before noticed, since the length of a sounding string varies inversely as the vibration-number, the question is equivalent to the following: what is the ratio between the vibration-numbers of two sounds forming an interval?

In the first place a few words must be said in regard to ratio. "Ratio is a mutual relation of two magnitudes of the same kind to one another in respect of quantity," or rather of "quántuplicity." (Euclid, Elements, V., def. 3.) It is immaterial which of the two magnitudes first receives the attention of the mind. It is also a matter of indifference which term of the ratio is regarded as compared with the other, whether the larger is compared with the smaller, or the smaller with the larger, provided one or the other manner is consistently adhered to during one and the same operation. It is usual to con-

sider the term first mentioned to be compared with the term last mentioned, as 2 to 3, i. e. 2 compared to 3. But if we wish to compare two ratios, as 2 to 3 and 5 to 7, to see which is the larger, or wider, we may either take the antecedents, 2 and 5, as standards, and so proceed to change the terms of the ratios until the antecedents are the same, and then compare the consequents (thus, 2:3=10:15 and 5:7=10:14, therefore 2:3 is wider because 15 is larger than 14); or we may regard the consequents, 3 and 7, as standards of comparison and compare 14:21 with 15:21. The latter method is more usual because ratios may be regarded as fractions. The consequents then become denominators, and the fractions are compared by reducing to a common denominator and comparing the numerators. (2/3= 14/21; 5/7 =15/21) But it would be just as legitimate to reduce the numerators to a common numerator, 10, and then to compare the denominators, 15 and 14.

Ancient arithmetic, like modern arithmetic, made a distinction between a ratio and the inverse ratio. When the greater of two numbers was compared with the less and so usually preceded it, the ratio (λόγος) was called πρόλογος. When the less was compared with the greater, the ratio was

called ὑπόλογος. The ancients then distinguished three kinds of ratio, according as the antecedent was greater than, was equal to, or was less than the consequent. Theo Smyrnaeus, De Musica 22, (Hiller, p. 7[illegible],): τῶν δὲ λόγων οἱ μέν εἰσι μείζονες, οἱ ἐλάττονες, οἱ ἴσοι.

Equal ratios are those in which the terms are equal. Of ratios where the first term is greater than the second, five kinds were distinguished: λόγοι πολλαπλάσιοι, ἐπιμόριοι, ἐπιμερεῖς, πολλαπλασιεπιμόριοι, and πολλαπλασιεπιμερεῖς.

A multiple ratio is one whose first term contains the second an exact number of times. A superparticular ratio is one whose first term contains the second once and also an aliquot part of the second. A superpartient ratio is one whose first term contains the second once and also more than one aliquot part of the second. Multiplex-superparticular and multiplex-superpartient ratios are like the last two kinds, but the first terms contain the second terms more than once, plus a fraction. Theo gives a sixth kind, (found also in Ptolemy Harm. I.v., Wallis, p. 10), namely λόγος ἀριθμοῦ πρὸς ἀριθμόν. Theo Smyrnaeus, p. 80, Ed. Hiller, (De Musica c. 28). It is not plain why this

should not be included under one of the other kinds. The example given is the ratio of 256 to 243. Superparticular ratios are named from the aliquot part of the smaller term necessary to make it equal to the greater term. Thus, λόγος ἐπίτριτος refers to the extra third of the smaller term, by which the greater exceeds the smaller. Any two consecutive numbers in the natural series except the first two, 1 and 2, form a superparticular ratio. The ratio 2:1 is included in the multiple ratios.

The ὑπόλογοι have the same names as those given for the πρόλογοι, but have the prefix ὑπο- added, as ὑποδιπλάσιος.

Now, musical intervals, as is well known, differ greatly in character, as well as in size. That is to say, notes standing at certain distances apart seem to bear a marked and peculiar relation to one another, which other notes do not possess. Some intervals are decidedly more pleasing than others, when both notes are sounded together. Others are so disagreeable that they may even become positively painful. Pleasing intervals are called consonant, unpleasing intervals are called dissonant. Consonance and dissonance are then

complementary terms. Intervals gain in consonance as they lose in dissonance. We might say that theoretically no interval is absolutely consonant or dissonant. In practice it is usual to make a classification of intervals into consonant intervals and dissonant intervals, and to assign every interval to one or other of these classes. The line of demarcation has varied from time to time. Intervals are called dissonant at one period and consonant at another. Modern music recognizes a greater number of consonant intervals than did ancient music. Many of the intervals now called imperfect consonances and imperfect dissonances were used in ancient music, not only in melody - note after note - but even in accompaniment, or harmony, - note against note - but they were all called dissonant intervals. Their function, when used in connection with consonant intervals, was to afford a contrast to the latter. The effects was similar to that of discords, employed in modern music to prevent the reiteration of concords from becoming a source of weariness instead of pleasure. It is described in the Aristotelian Problems XIX, 39: εὐφραινόμεθα μᾶλλον τῷ τέλει ἢ λυπούμεθα ταῖς πρὸ τοῦ τέλους διαφοραῖς. Cf. Westphal, Die Musik des Griechischen Alterthums, p. 64, where the translation

of this passage is by no means too literal. The reason that modern music admits so many intervals to the ranks of consonances is that the ear has become accustomed to them through constantly hearing them used in simultaneous harmony. The general notion of consonance and dissonance is, however, the same now as it was in classical times.

All the consonances recognized among the Greeks at any period of ancient music may be reduced to three principal consonant intervals, the Octave, the Fifth, and the Fourth. All other consonant intervals may be derived from these three and consist of two or more of them added together. It is true that the Octave is equal to the sum of the Fifth and Fourth, but for certain reasons it is more proper to regard the Fifth and the Fourth as parts of the Octave, than the Octave as the result of compounding the Fifth and the Fourth.

Let us then see what ratios were discovered to belong to these consonances, called by the Greeks διὰ πασῶν, διὰ πέντε and διὰ τεσσάρων, respectively, and how they are identified with the corresponding modern intervals.

Pythagoras was apparently the first among the Greeks to discover the numerical relations existing between musical sounds

1.
Diogenes Laertius VIII, 11. Bryennius I. i., p. 361, Ed. Wallis. We have the following statement of Xenocrates, as quoted by Heraclides Ponticus. The passage is found in an excerpt from the Εἰσαγωγὴ Μουσική of Heraclides given by Porphyry, Commentarius in Ptolemaeum, I., c. iii., init. p. 213, Ed. Wallis: Πυθαγόρας, ὥς φησι Ξενοκράτης, εὗρε καὶ τὰ ἐν μουσικῇ διαστήματα οὐ χωρὶς ἀριθμοῦ τὴν γένεσιν ἔχοντα· ἔστι γὰρ σύγκρισις ποσοῦ πρὸς ποσόν. Cp. Macrobius, Commentarius in Somnium Scipionis II i., 8 fol. and Jan's note. Iamblichus in his Life of Pythagoras says that he was taken from Egypt to Babylon and there learned the science of number and of music.

Pythagoras undoubtedly used a single string or what amounts to practically the same thing - two strings of equal length tuned in unison - and obtained his ratio by means of a moving bridge, such that it would not add to the original tension of the string, or by stopping off various lengths on a long string furnished with a finger-board. The lyre would

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1. Gevaert, Histoire et Théorie de la Musique de l'antiquité' I., p. 74; Westphal, Rhythmik and Harmonik, p. 62; Musik, p. 176.

be unsuitable for performing these experiments because each string is capable of producing only one note. There is no method by which a string can be shortened. But the Egyptians had instruments with very long strings and with finger-boards, and Pythagoras may easily have been acquainted with these.

The ratios determined by Pythagoras for the three consonances above mentioned were 1:2 for the Octave, that is, a string whose length is 2 sounds a note an Octave lower than one whose length is 1; 2:3 for the Fifth, and 3:4 for the Fourth. These ratios were no doubt obtained by direct observation. Other ratios may easily have been obtained for other intervals, either directly from the string, or indirectly by combining the ratios. Thus, the consonant intervals of the Twelfth and Double Octave were probably found to depend on the ratios 1:3 and 1:4 respectively. The Tone, which is the difference in width between the Fifth and the Fourth, was certainly regarded as dependent on the ratio 8:9, but it is probable that this ratio was often deduced from those already obtained, and was not directly observed from the lengths of string.

The Tone has always held a very prominent position in

musical theory owing to two facts. The first is that it is the difference of two consonances and may be tuned with great accuracy by means of them. The second is that the ratio 8:9 taken six times ( i.e. 262144:531441) differs only by a very small ratio from the ratio 1:2 (= 262144:524288). This small difference, called the Pythagorean Comma, is only a little larger than the ordinary Comma (80:81), an interval which is neglected in the tuning of modern keyed instruments. Owing to these two facts, which we may almost call arithmetical accidents, the Tone has received the attention of musical theorists to an extent which is not warranted by the musical qualities of the interval. Other intervals, approaching it in size, and also called tones, like the interval 7:8 (supersecond or septimal second) and 9:10 (Minor tone) are used in modern music quite as frequently as the Major tone 8:9. But because the Major Tone was the only difference between intervals recognized by the Greeks to be consonant which was not itself consonant, it was given an important part in the formation of theoretical scales. The facility with which it may be tuned would also undoubtedly have great influence in causing it to appear in scales as actually tuned on the lyre. But if it is possible to draw conclusions by analogy from

facts presented to us in the history of modern theory, it is very probable that this Major Tone impersonated, so to speak, other intervals similar to it in size. It often happens that the real interval, the interval as actually sung,, or, perhaps it would be better to say, the interval as the singer desires to sing it, is mistaken for some other interval, because it is approximately equal to it. It is not at all unlikely that the same thing took place in ancient theory. The scale which is constructed in the Timaeus is artificial in this respect.

This passage (Plato Tim. 35 B.fol.) is perhaps the earliest in which the consonant ratios are mentioned. It is to be observed, however, that there is nowhere any reference to music in the text. The scale is essentially a theoretical one. The procedure is as follows: First the double geometrical quaternion or tetractys of the Pythagoreans is formed by joining to unity the first three powers of 2 and of 3, thus, 1,2,3,4,9,8,27. This tetractys may be arranged so as to exhibit the two branches consisting of powers of 2 and 3 respectively, by making a Lambda as follows:

1

2 3

4 9

8 27

The left branch contains the double intervals (διπλάσια διαστήματα) and the right one the triple intervals (τριπλάσια διαστήματα). Then between the terms of every interval the harmonical and the arithmetical means are inserted. In this way two series are obtained:

1, 4/3, 3/2, 2, 8/3, 3, 4, 16/3, 6, 8 and

1, 3/2, 2, 3, 9/2, 6, 9, 27/2, 18, 27.

The succession of intervals in the first series is 4/3, 9/8, 4/3, thrice repeated, and in the second it is 3/2, 4/3, 3/2. The explanation of these series is held by commentators to be that they refer to musical scales. The ratios 3/2 and 4/3 will then correspond to the consonances of the Fifth and the Fourth, and their product (3/2x4/3=2/1) to the sum of these intervals, the Octave. The last step is that by which every interval of 3:4 was filled up with intervals of 8:9, as many as are contained in 3:4, and with λείμματα, which are as 243 to 256. Scales are therefore formed in which each note differs from its neighbors by an interval of either

8:9 or 243:256, since every Fifth may be resolved into a Fourth and a Tone (8:9). Every step is therefore either an Interval of a Tone or of a Leimma. The scale formed on the binary branch of the tetractys has a compass of three octaves, each octave containing five Tones and two Leimmata. The scale formed on the ternary branch has a compass of three Twelfths. Each Twelfth or Dodecachord is of the form: Nete diezeugmenon - Mese - Hypate meson - Proslambanomenus (the names are those of the Perfect System). The intervals are , Fifth, Fourth, Fifth, and together form an Octave, like the Octave of the binary scale, plus the interval of a Fifth towards the bass, Hypate - Proslambanomenus. Each of the three Dodecachords then contains 8 Tones and 3 Leimmata.

The fact that the compass of each of these scales is much larger than that of any scale described in the musical treatises goes far towards showing that they are not to be regarded as actual musical scales.[1] In the Dodecachords there is the further objection that each of them is in a different key. In other words, the scale passes into two new keys.

The question then naturally occurs, Are these scales musical scales at all in the modern sense of the word musical? Do they not rather belong to the music of numbers (ἡ ἐν

1. Westphal, die Musik, p.178, note.

ἀριθμοῖς μουσική)?

The ancient commentators on the passage themselves support the view. It is admitted by Adrastus, quoted by Theo Smyrnaeus, De Musica, c. 13, p. 64, Ed. Hiller, that the compass of scales actually used in music falls far short of that of the scale described by Plato, of which the length is three Octaves and a Major Sixth (= three Twelfths), but it is pointed out that it is necessary to extend the scale into cubic numbers, because they represent solids. In any case, the scale, even if it is purely an imaginary musical scale, seems to have been suggested by some real scale, and may be illustrated and explained by supposing such a scale. We may safely see in the intervals between the terms of the series, references to the ratios associated with musical intervals.

We have in Euclid's Sectio Canonis the first explicit statement of these ratios.

The first ten theorems are purely mathematical, the remaining nine are musical in the narrower sense. In 11 he proves that the interval διὰ πασῶν is multiple; in 12 that the διὰ τεσσάρων and διὰ πέντε are each superparticular; in 13 the ratio for the διὰ πασῶν is proved to be 2:1; in 14 the ratios for the διὰ τεσσάρων

and διὰ πέντε are shown to be respectively 3:2 and 4:3; and in 15 the ratio for the διὰ πασῶν καὶ διὰ πέντε (Twelfth) is proved to be 3:1. The 16th, 17th, 18th, and 19th are on the Tone (τόνος), whose ratio is 9:8. The largest consonant interval, it should be noticed, is the Twelfth (3:1 ). This cannot, however, be taken as evidence that in Euclid's time, the gamut had not expanded beyond the Dodecachord, because in the 11th Proposition, in which it is proved that the ratio for the Octave is multiple, mention is made of Νήτη Ὑπερβολαίων, Μέση, and Προσλαμβανόμενος, notes which belong to the Perfect System of two Octaves' extent and stand at intervals of an Octave from one another.

The Aristotelian Problems, even if not the work of Aristotle, are thought to be not much later than his time.

The following are the most important references to the consonant interval ratios contained in the 19th section entitled Ὅσα περὶ Ἁρμονίαν:

Problems XIX, 39: ἁρμόζουσι δὲ ἐν τῇ διὰ πασῶν συμφωνίᾳ, ὅτι, καθάπερ ἐν τοῖς μέτροις οἱ πόδες ἔχουσι πρὸς αὑτοὺς λόγον ἴσον πρὸς ἴσον ἢ δύο πρὸς ἓν ἢ καί τινα ἄλλον

αὐτοὶ καὶ οἱ ἐν τῇ συμφωνίᾳ φθόγγοι λόγον ἔχουσι κινήσεως πρὸς αὑτούς

In this passage the existence of ratios is affirmed. It is important to notice that ratio is here derived, not from numbers expressing the relative lengths of sounding strings or pipes, but from the vibrations of strings or perhaps of the air.

Problems XIX,35: διὰ τί ἡ διὰ πασῶν καλλίστη συμφωνία; ἤ, ὅτι ἐν ὅλοις ὅροις οἱ ταύτης λόγοι εἰσίν, οἱ δὲ τῶν ἄλλων οὐκ ἐν ὅλοις; ἐπεὶ γὰρ διπλασία ἡ νήτη τῆς ὑπάτης, οἷα ἡ νήτη δύο, ἡ ὑπάτη ἕν, καὶ οἷα ἡ ὑπάτη δύο, ἡ νήτη τέσσαρα, καὶ ἀεὶ οὕτως· τῆς δὲ μέσης ἡμιόλια. τὸ γὰρ διὰ πέντε ἡμιόλιον οὐκ ἐν ὅλοις ἀριθμοῖς ἐστίν. οἷον γὰρ ἐν τῷ ἐλάττονι τὸ μεῖζον τοσοῦτόν τε καὶ ἔτι τὸ ἥμισυ. ὥστε οὐχ ὅλα πρὸς ὅλα συγκρίνεται ἀλλὰ ἐπιμόρια μέρη. ὁμοίως δὲ καὶ ἐν τῷ διὰ τεττάρων ἔχει· τὸ γὰρ ἐπίτριτόν ἐστι. τὸ γὰρ ἐπίτριτόν ἐστιν ὅσον τὰ μεῖον ὃ καὶ ἔτι ἓν τῶν τεττάρων ἐπίτριτόν ἐστιν.

τι μὲν δὲ ἄγειν τοιοῦτον is suggested in the critical notes in Bekker's Edition. The last sentence here quoted is evidently corrupt.

This passage gives the proper ratios for the three consonant intervals διὰ πασῶν, διὰ πέντε, and διὰ τεσσάρων. The scale implied has the three notes mentioned, Nete, Mese, and Hypate, situated as follows with reference to one another:

| | | | |
|---|---|---|---|
| 12 | νήτη | } διὰ πέντε | } διὰ πασῶν |
| 8 | μέση | } διὰ τεσσάρων | |
| 6 | ὑπάτη | | |

Problems XIX. 41:

διὰ τί δὶς μὲν δι' ὀξειῶν ἢ δὶς διὰ τεττάρων οὐ συμφωνεῖ, δὶς διὰ πασῶν δέ; ἢ ὅτι τὸ μὲν διὰ πέντε ἐστὶν ἐν ἡμιολίῳ λόγῳ, τὸ δὲ διὰ τεττάρων ἐν ἐπιτρίτῳ;

and a few lines on: τὸ δὲ διὰ πασῶν ἐπειδή ἐστιν ἐν διπλασίῳ λόγῳ, — — κτέ.

Problems, XIX.50:

διὰ τί ἴσων πίθων καὶ ὁμοίων ἐὰν μὲν ὁ ἕτερος κενὸς ᾖ, ὁ δὲ ἕτερος εἰς τὸ

ἥμισυ ἡμικένωτος, διὰ πασῶν συμφωνεῖ ἡ ἠχώ; ἢ ὅτι διπλασία γίνεται καὶ ἡ ἐκ τοῦ ἡμίσεος καὶ ἡ ἐκ τοῦ κενοῦ; τί γὰρ διαφέρει τοῦτο ἢ ἐπὶ τῶν συρίγγων; δοκεῖ γὰρ ὁ θάττων κίνησις ὀξυτέρα εἶναι, ἐν δὲ μείζοσι βραδύτερον ὁ ἀὴρ ἀπαντᾷ, καὶ ἐν διπλασίοις τοσούτῳ, καὶ ἐν τοῖς ἄλλοις ἀνὰ λόγον. συμφωνεῖ δὲ διὰ πασῶν καὶ ὁ διπλασίων ἀσκὸς πρὸς τὸν ἥμισυν.

Problems XIX.23:

διὰ τί διπλασία ἡ νήτη τῆς ὑπάτης; ἢ πρῶτον μὲν ὅτι ἐκ τοῦ ἡμίσεος ἡ χορδὴ ψαλλομένη καὶ ὅλη συμφωνοῦσα [συμφωνοῦσι ?] διὰ πασῶν; ὁμοίως δὲ ἔχει καὶ ἐπὶ τῶν συρίγγων· ἡ γὰρ διὰ τοῦ μέσου τῆς σύριγγος τρήματος φωνὴ τῇ δι' ὅλης τῆς σύριγγος συμφωνεῖ διὰ πασῶν. ἔτι ἐν τοῖς ἄλλοις τῷ διπλασίῳ διαστήματι λαμβάνεται τὸ διὰ πασῶν, καὶ οἱ αὐλοτρῦπαι οὕτω λαμβάνουσιν. [ὁμοίως δὲ καὶ τὸ διὰ πέντε τῷ ἡμιολίῳ] ὅτι οἱ τὰς σύριγγας ἁρμοττόμενοι εἰς μὲν τὴν ὑπάτην ἄκραν

13 τὸν κενὸν ἐναπολείπουσι, τὴν δὲ ἑτέραν μέχρι τοῦ ἡμίσεος ἀναπληροῦσιν. ὁμοίως δὲ καὶ διὰ πέντε τῷ ἡμιολίῳ καὶ τὴν διὰ τεσσάρων τῷ ἐπιτρίτῳ διαστήματι λαμβάνουσιν. ἔτι δὲ καὶ ἐν τοῖς τριγώνοις ψαλτηρίοις ⟨χορδαὶ (?)⟩ τῆς ἴσης ἐπιτάσεως γινομένης συμφωνοῦσι διὰ πασῶν, ἡ μὲν διπλασία οὖσα, ἡ δὲ ἡμίσεια τῷ μήκει.

In the above interesting problem, I have taken the liberty of enclosing ὁμοίως κτέ in l.10 in brackets. These words have crept in from l.14 below where they are appropriate. The upper passage is best without them.

Theo Smyrnaeus, Expositio Rerum Mathematicarum ad Legendum Platonem Utilium, p. 56, Ed. Hiller. (De Musica c. 12):

τοὺς δὲ συμφωνοῦντας φθόγγους ἐν λόγοις τοῖς πρὸς ἀλλήλους πρῶτος ἀνευρηκέναι δοκεῖ Πυθαγόρας, τοὺς μὲν διὰ τεσσάρων ἐν ἐπιτρίτῳ, τοὺς δὲ διὰ πέντε ἐν ἡμιολίῳ, τοὺς δὲ διὰ πασῶν ἐν διπλασίῳ, καὶ τοὺς μὲν διὰ πασῶν καὶ διὰ τεσσάρων ἐν λόγῳ τῷ η′ πρὸς γ′ ὅς ἐστι πολλαπλασιεπιμερής, διπλάσιος γὰρ καὶ διεπίτριτός ἐστι, τοὺς δὲ διὰ πασῶν καὶ διὰ πέντε ἐν λόγῳ τριπλασίῳ, τοὺς δὲ δὶς διὰ

μάτων ἐν τετραπλασίῳ, καὶ τῶν ἄλλων ἡρμοσμένων τοὺς μὲν τὸν τόνον περιέχοντας ἐν ἐπογδόῳ λόγῳ, τοὺς δὲ τὸ νῦν λεγόμενον ἡμιτόνιον, τότε δὲ δίεσιν, ἐκ ἀριθμοῦ λόγῳ πρὸς ἀριθμὸν τῷ τῶν σνϛ´ πρὸς σμγ´.

The manner of arriving at these ratios is described in the following passage: The author has just mentioned some of the methods employed by Pythagoras. He seems to have compared the lengths and thicknesses of strings, and their tension as shown by the turning of the pegs or by the hanging of weights, the bore of the cavities of wind instruments, the force of the breath, and the masses and weights of discs and vessels.

Theo Smyr. Expositio, p. 57, l. 11, Ed. Hiller (De Musica, c. 12): ἀρκείτω δ' ἡμῖν ἐν τῷ παρόντι διὰ τοῦ μήκους τῶν χορδῶν δηλῶσαι καὶ τοῦ λεγομένου κανόνος. τῆς γὰρ ἐν τούτῳ μιᾶς χορδῆς καταμετρηθείσης εἰς τέσσαρα ἴσα, ὁ ἀπὸ τῆς ὅλης φθόγγος τῷ μὲν ἀπὸ τῶν τριῶν

μερῶν ἐν λόγῳ γενόμενος ἐπιτρίτῳ, συμφωνήσει διὰ τεσσάρων, τῷ δὲ ἀπὸ τῶν δύο, τουτέστι τῷ ἀπὸ τῆς ἡμισείας, ἐν λόγῳ γενόμενος διπλασίῳ, συμφωνήσει διὰ πασῶν, τῷ δὲ ἀπὸ τοῦ τετάρτου μέρους, γενόμενος ἐν λόγῳ τετραπλασίῳ, συμφωνήσει δὶς διὰ πασῶν. ὁ δὲ ἀπὸ τῶν τριῶν μερῶν φθόγγος πρὸς τὸν ἀπὸ τῶν δύο γενόμενος ἐν ἡμιολίῳ, συμφωνήσει διὰ πέντε, πρὸς δὲ τὸν ἀπὸ τοῦ τετάρτου μέρους, γενόμενος ἐν λόγῳ τριπλασίῳ, συμφωνήσει διὰ πασῶν καὶ διὰ πέντε.

The following diagram will illustrate the operations described:

Acute.

| | | | |
|---|---|---|---|
| λ. τετραπλάσιος / δὶς διὰ πασῶν | | 1 | λόγος τριπλάσιος / διὰ πασῶν καὶ διὰ πέντε |
| | λόγος ἡμιόλιος / διὰ πέντε | 2 | |
| | λόγος ἐπίτριτος / διὰ τεσσάρων | 3 | λόγος διπλάσιος / διὰ πασῶν |
| | | 4 | |

Grave.

Theo then states that the tetractys consisting of 1,2,3, and 4 exhibits every consonant interval. It contains the λόγος ἐπίτριτος, λόγος ἡμιόλιος, λόγος διπλάσιος, λόγος τριπλάσιος and λόγος τετραπλάσιος.

He next shows how the ratios may be derived from vessels of different sizes. Cp. Arist. Probl. XIX. 23.

Theo Smyrn. p. 59, Ed. Hiller:

ἴσων γὰρ ὄντων καὶ ὁμοίων πάντων τῶν ἀγγείων, τὸ μὲν κενὸν ἐάσας, τὸ δὲ ἥμισυ ὑγροῦ ⟨πληρώσας⟩ ἐψόφει ἑκατέρῳ, καὶ αὐτῷ ἡ διὰ πασῶν ἀπεδίδοτο συμφωνία· θάτερον δὲ πάλιν τῶν ἀγγείων κενὸν ἐῶν εἰς θάτερον τῶν τεσσάρων μερῶν τὸ ἓν ἐνέχει, καὶ κρούσαντι αὐτῷ ἡ διὰ τεσσάρων συμφωνία ἀπεδίδοτο, ἡ δὲ διὰ πέντε, ⟨ὅτε⟩ ἓν μέρος τῶν τριῶν συνεπλήρου, οὔσης τῆς κενώσεως πρὸς τὴν ἑτέραν ἐν μὲν τῇ διὰ πασῶν ὡς β΄ πρὸς ἕν, ἐν δὲ τῇ διὰ πέντε ὡς γ΄ πρὸς β΄, ἐν δὲ τῇ διὰ τεσσάρων ὡς δ΄ πρὸς γ΄.

or by the division of st ings:

εἰς ὁμοίως καὶ κατὰ τὰς διαλήψεις τῶν χορδῶν θεωρεῖται, ὡς προείρηκα, ἀλλ' οὐκ ἐπὶ μιᾶς χορδῆς, ὡς ἐπὶ τοῦ κανόνος, ἀλλ' ἐπὶ δυοῖν· δύο γὰρ ποιήσας ὁμοτόνους ὅτε μὲν τὴν μίαν αὐτῶν διαλήψει μέσην πιέσας, τὸ ἥμισυ πρὸς τὴν ἑτέραν συμφωνίαν τὴν διὰ πασῶν ἐποίει· ὅτε δὲ τὸ τρίτον μέρος ἀπολαμβάνοι, τὰ λοιπὰ μέρη πρὸς τὴν ἑτέραν τὴν διὰ πέντε συμφωνίαν ἐποίει· ὁμοίως δὲ καὶ ἐπὶ τῆς διὰ τεσσάρων· καὶ γὰρ ἐπὶ ταύτης μιᾶς τῶν χορδῶν ἀπολαβὼν τὸ τέταρτον μέρος, τὰ λοιπὰ μέρη πρὸς τὴν ἑτέραν συνῆπτεν.

Panpipes also give the same ratios. So too, it is stated, do the weights attached to strings, but, as was noted at p.62, such weights will not give the ratios sought for but the duplicate ratios corresponding to them. In order to pro-



duce any interval the stretching weights must be to one anoth-

er as the squares of the simple ratios which obtain between lengths of string or between vibrations.

Once more, the ratios are observable in flutes ( αὐλοί ) according to the disposition of the finger-holes. The measurements are made from the upper end downward to the holes. The determination of the ratio in this manner is described at the beginning of the 13th Chapter of Theo's work De Musica (or Hiller, p. 60,61.) The complete correspondence both of the ratios with the intervals as determined by the ear, and of the intervals with the ratios as observed, is stated in the words of Adrastus, as quoted by Theo:

*Theo Smyrnaeus, Ex Mus. c. 13. p. 61. l. 20 Ed Hiller.*

ἀλλὰ γὰρ ὅτι τούτοις τοῖς εἰς τὴν εὕρεσιν τῶν συμφωνιῶν ὀργάνοις καὶ κατὰ τοὺς λόγους προπαρασκευασθεῖσιν ἡ αἴσθησις καταμαρτυρεῖ, τῇ δὲ αἰσθήσει προσληφθείσῃ ὁ λόγος ἐφαρμόζει.

The following paragraphs are devoted to showing that the ratios may be compounded so as to produce the sum of the corresponding intervals, and that consistent results may be ob-

tained from arithmetical operations performed on the ratios.

We turn now to the consideration of the quaternions, which were regarded by the Pythagoreans as endowed with peculiar properties.

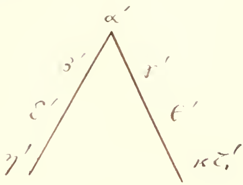
Theo has already mentioned the tetractys which is composed of the first four natural numbers and so constitutes the decade - ἡ τῆς δεκάδος τετρακτύς (See p. 82 following.)

He returns to it in chapter 37.

Theo Smyrn. De Mus. c. 37;p.93, Ed. Hiller:

ἐπειδὴ πάντες οἱ τῶν συμφωνιῶν εὑρέθησαν λόγοι, καθὰ δέδεικται, ἐν τῇ τῆς δεκάδος τετρακτύι, καὶ περὶ τούτων πρότερον λεκτέον. τὴν μὲν γὰρ τετρακτὺν συνέστησεν ἡ δέκας. ἓν γὰρ καὶ β' καὶ γ' καὶ δ' ι'. α' β' γ' δ'. ἐν δὲ τούτοις τοῖς ἀριθμοῖς ἔστιν ἥ τε διὰ τεσσάρων συμφωνία ἐν ἐπιτρίτῳ λόγῳ καὶ ἡ διὰ πέντε ἐν ἡμιολίῳ καὶ ἡ διὰ πασῶν ἐν διπλασίῳ καὶ <ἡ> δὶς διὰ πασῶν ἐν τετραπλασίῳ.

Theo takes up next the two-branched quaternion which Plato discusses in the Timaeus (35 B.fol.) (See p. 73 ) One branch is formed of odd numbers, the other of even numbers. Unity is both even and odd and is common to the two branches . The following is the arrangement in which the branches are shown to converge:




Theo Smyrn. p.95,Ed. Hiller:

ἐν τούτοις τοῖς ἀριθμοῖς ⟨αἱ⟩ τελειότεραι τῶν συμφωνιῶν εὑρίσκονται λόγοι· συμπεριείληπται δὲ αὐτοῖς καὶ ὁ τόνος.

This tetractys had also the merit of consisting of seven terms - a number which differs from the other nine members of the decade in that it neither generates another nor is generated by another - in other words,it is prime and is not a factor of any of the first ten numbers. (Theo. Smyrn. p. 103, Ed. Hiller.)

Plutarch devotes considerable space to this tetractys in his De Animae Procreatione in Timaeo Platonis. After commenting on the Pythagorean tetractys, the number 36, (De Anim. Procr. c.xxx=1027 F, then c. xi=1017 D. For this order see Paul Tannery in the Revue des Études Grecques, vii, p. 209.), Plutarch confers even higher praise on the double tetractys set forth by Plato in the Timaeus. In it, as was noticed above (p.73 ) the left branch consists of four powers of 2, and the right, of four powers of 3.

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $2^0 = 3^0$ | | | | or | | | | 1 | | |
| | | $2^1$ | | $3^1$ | | | | | | 2 | | 3 | |
| | $2^2$ | | | | $3^2$ | | | | 4 | | | | 9 |
| $2^3$ | | | | | | $3^3$ | | 8 | | | | | 27 |

The Lambda-like arrangement Plutarch attributes to Crantor (1027 D). The advantage gained is that like powers may be more easily compared for purposes of multiplication and addition. By adding like powers we obtain 5, 13, and 35. These numbers, it is stated, were significant to the Pythagoreans of various musical conceptions. The first, 5, they called τρόφον (which is explained by the clause ὅπερ ἐστὶ ψόφον ), on the supposition that the "fifth of the intervals of the Tone" is the first which is audible. It is not easy to see

why the smallest audible interval, which seems to be the meaning of the words πρῶτον φθεγκτόν, should be called τροφόν, unless it be that they regarded it as the 'food' or material out of which all intervals are built up.

The whole passage runs as follows:

Plut. De Anim. Procr. xii., 1017 F.:

τούτων γὰρ τῶν ἀριθμῶν οἱ Πυθαγορικοί, τὰ μὲν ε', τροφόν, ὅπερ ἐστὶ φθόγγον, ἐκάλουν, οἰόμενοι τῶν τοῦ τόνου διαστημάτων πρῶτον εἶναι φθεγκτὸν τὸ πέμπτον·

For τροφόν I offer the conjecture ψόφον. The explanatory clause, ὅπερ ἐστὶ φθόγγον, seems to support this view.

The passage continues:

τὰ δὲ τρισκαίδεκα, λεῖμμα, καθάπερ Πλάτων, τὴν εἰς ἴσα τοῦ τόνου διανομὴν ἀπογινώσκοντες·

It was a cardinal doctrine of the Pythagoreans that the Semitone ( called by them both Leimma and Diësis, see Theo, p. 55, l. 13, Ed. Hiller.) was not the exact half of a

1.
Tone.

Why 13 was identified with the Leimma is not st-ted in this passage, but in Chapter xiv, in which the Tone is denoted by 27, the grounds for calling 13 the Leimma may be seen. We shall recur to this point later on (p.95).

The third number obtained by adding like powers of 2 and 3 is 35, the sum of 8 and 27. This the Pythagoreans called ἁρμονία. Besides equalling the sum of the first two cubes it consists of four numbers 6,8,9, and 12, which include the arithmetical and harmonical progressions. 6,9,12, is in Arithmetical Progression and 6,8,12, is in Harmonical Progression.

Plut. De Anim. Procr. xii., 1017 F. cont.

τὰ δὲ πέντε καὶ τριάκοντα, ἁρμονίαν, ὅτι συνέστηκεν

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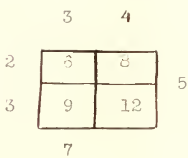
1. Cp. Plut. De Anim. Procr. c. xvii, 1020 F.

τοῦτο [i.e. τὸν τόνον] οἱ μὲν ἁρμονικοὶ δίχα τεμνόμενον οἴονται δύο διαστήματα ποιεῖν, ὧν ἑκάτερον ἡμιτόνιον καλοῦσιν· οἱ δὲ Πυθαγορικοὶ τὴν μὲν εἰς ἴσα τομὴν ἀπέγνωσαν αὐτοῦ, τῶν δὲ τμημάτων ἀνίσων ὄντων λεῖμμα τὸ ἔλαττον ὀνομάζουσιν, ὅτι τοῦ ἡμίσεως ἀπολείπει — κτέ

and the beginning of chapter xviii.

ἐκ δυεῖν κύβων πρώτων ἀπὸ ἀρτίου καὶ περιττοῦ γεγονότων, ἐκ τεσσάρων δὲ ἀριθμῶν, τοῦ ς' καὶ τοῦ η' καὶ τοῦ θ' καὶ ιβ', τὴν ἀριθμητικὴν καὶ τὴν ἁρμονικὴν ἀναλογίαν περιεχόντων.

To illustrate the proposition a rectangle is constructed whose sides are as 5 and 7. The whole area will then be 35. Lines parallel to the sides are then drawn, cutting the side whose length is 5 into two parts, 2 and 3, and cutting the side whose length is 7 into two parts, 3 and 4. The whole rectangle will then be divided into four compartments, whose areas will be 6,8,9, and 12 (sum=35),




and these will include the ratios of the 'first consonances.' Plut. De Anim. Procr. xii. 1018 A. fin.:

τὸ δὲ ὅλον παραλληλόγραμμον τριάκοντα καὶ πέντε τοὺς τῶν συμφωνιῶν τῶν πρώτων λόγους ἐν τοῖς τῶν χωρίων ἀριθμοῖς, εἰς ἃ διῄρηται, περιέχον. τὰ μὲν οὖν ἓξ καὶ ὀκτὼ

ἐπίτριτον ἔχει λόγον, ἐν ᾧ τὸ διὰ τεσσάρων· τὰ δὲ ἓξ καὶ ἐννέα, τὸν ἡμιόλιον, ἐν ᾧ τὸ διὰ πέντε· τὰ δὲ ἓξ καὶ ιβ', τὸν διπλάσιον, ἐν ᾧ τὸ διὰ πασῶν· ἔνεστι δὲ καὶ ὁ τοῦ τόνου λόγος ἐπόγδοος ὢν ἐν τοῖς ἐννέα καὶ ὀκτώ. διὰ τοῦτο καὶ ἁρμονίαν τὸν περιέχοντα τοὺς λόγους τούτους ἀριθμὸν ἐκάλεσαν.

In the next chapter (xiii) corresponding numbers in the two branches of the Platonic tetractys (it is so called at xiv fin. 1019 B.) are multiplied together. 6,13, and 216 are obtained, which are of course, the first, second, and third power of 6. Of these, 36 is at once the square of 6, the triangle[1] of 8, and two parallelograms - 9x4 and 12x3. In these we find once more the numbers 6,8,9, and 12 and the ratios of the consonances.

Plut. De Anim. Procr. xiii, 1018 D:

ἔστι γὰρ τὰ δώδεκα πρὸς μὲν τὰ ἐννέα διὰ τεσσάρων, ὡς νήτη πρὸς μέσην· πρὸς

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1. For triangular numbers see Theo Smyrn. p. 37, Ed. Hiller.

36 = 1+2+3+4+5+6+7+8.

δὲ τὰ ὀκτώ, διὰ πέντε, ὡς νήτη πρὸς παραμέσην· πρὸς δὲ τὰ ς΄, διὰ πασῶν, ὡς νήτη πρὸς ὑπάτην.

As it stands in the text, the passage would give:

| | |
|---|---|
| 12 | νήτη |
| 9 | μέση |
| 8 | παραμέση |
| 6 | ὑπάτη |

But Paramese is made to stand at a wider interval from Nete than does Mese. In no scale, however, do we find the note called Paramese of graver pitch than Mese. Paramese is always nearer to Nete, the highest note of the Octachord, than is Mese, irrespectively of the width of the interval Mese - Nete - We must consequently change μέσην to παραμέσην and παραμέσην to μέσην. Emended in this manner, the passage gives us the scale:

| | | |
|---|---|---|
| διὰ τεσσάρων (12–9) | 12 | νήτη |
| | 9 | παραμέση |
| διὰ πέντε (12–8) | 8 | μέση |
| διὰ πασῶν (12–6) | 6 | ὑπάτη |

Returning to the double geometrical quaternion 1,2,3,4,

9,8,27 in chapter xiv, Plutarch comments on the property peculiar to the last of the series, 27, that it is equal to the sum of its predecessors. The Pythagoreans assign the Tone to this number. The Leimma is thus a little less than one-half of the Tone.

Plutarch does not at this place explain how 27 and 13 came to be associated with the Tone and the Leimma, but in chapter xviii, where he shows how the interval of a Fourth must be divided to give the two whole Tones and the Leimma left over, he makes it plain that 13 is nothing more nor less than the arithmetical difference between 256 and 243. These numbers are the smallest numbers which can express exactly the difference between the ratios 3:4 and $\hat{8}:\hat{9}$. But as Theo Smyrn. says (p.69, l. 7, Hiller), in discussing this same division of the Fourth, there is nothing to prevent one from using other numbers to express the ratio of the Leimma - such as 512 to 486. But since 256:243 is the ratio reduced to its lowest terms, the Pythagoreans took the arithmetical difference of these figures to represent the interval. In the same way 27 represents the Tone. For, take two numbers whose ratio is 3:4. Let the smaller number stand for the graver sound. Then, in order that this smaller number may be twice increased by an eighth

part of itself (which will correspond to raising the lower note two Tones), it is necessary that there be two factors 8 in the number. Since the two numbers are to be in the ratio 3:4, 3 must also be a factor of the smaller number. Therefore the number is 3x8x9=192, and the series runs:

| (grave) | Tone | Tone | Leimma | (acute) |
|---|---|---|---|---|
| 192 | 216 | 243 | 256 | |

256 - 243 = 13, the Leimma.

243 - 216 = 27, one of the Tones.

This is essentially the method employed in chapter xviii to explain the passage in the Timaeus (36 B.) where the Creator "filled up all the intervals of 4/3 with that of 9/8, leaving in each a fraction over." But in Plato small numbers represent acute sounds, and the scale runs downwards, whereas in Plutarch the reverse is the case. In Plato's scale the Leimmata are at the grave end of the tetrachords - a position which they hold in normal Greek scales; according to Plutarch's arrangement in this passage, the Leimma stands above the Tones. The same set of figures can illustrate these two situations according as they stand for lengths of string or for vibrations or tension. Theo Smyrnaeus uses the same figures to demonstrate the proposition that the Fourth is not equal to two

Tones, and a half, but with him the more usual course is adopted of making acute sounds correspond to small numbers. See Theo Smyrn. p. 65, l. 10, fol., and p. 67 l. 16 fol. Ed. Hiller.

In the following chapter (XIX) Plutarch mentions other proportions which are used to illustrate the composition of the Fourth. When the Leimma is the middle of the three intervals, the proportion runs:

| | Tone | Leimma | Tone | |
|---|---|---|---|---|
| 216 | | 243 | 256 | 288 |

It is, of course, somewhat illogical to identify an interval with the absolute difference between the terms of the ratio, instead of with the relative difference. The absolute difference is variable. For the Tone it is now 24, now 27, and now 32, as shown in the diagrams given above. For a dissonant interval, like the Leimma, the difference may always be some number or some multiple thereof ( as in the case of the Leimma it is 13, or a multiple of 13); but in the case of consonances and the more perfect (i.e. consonant) of the dissonant intervals the absolute or arithmetical difference may be almost any number whatsoever, and when the ratio is reduced to its lowest terms, it will generally be unity.

Since the Leimma was as a matter of practice the lowest

of the three intervals into which every 'standing'[1] tetrachord was divided when in a scale of the Diatonic genus and of the variety or χρόα called the high-pitched (διάτονον σύντονον, or perhaps, more accurately, διάτονον διτονιαῖον), we may imagine experimenters to have proceeded somewhat as follows. Let the sound produced by the whole length of a string be the lowest note of a Fourth. The upper note of the interval will then be produced by three-quarters of the string. In order to descend by whole tones from this upper note, it is necessary twice in succession to increase the length of the string by an eighth part of itself. The three-fourths of the string must then be multiplied by 9/8 to give the length which will sound the second (descending) note, and the result by 9/8 again to give the third note. We will have 3/4 x 9/8 x 9/8 = 243/256 for the length used for this note. If the whole string is now divided into 256 equal parts, the first 13, when 'stopped off' by the finger, will give the interval of a Leimma, the next 27 will give the interval of a Tone, and so will the next 24. In this way the Pythagorean numerical

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1. Φθόγγοι ἑστῶτες are those which do not vary in pitch with the genus and color. They form the frame-work of every scale.

values for the Leimma and Tone may be practically illustrated.

If five parts are measured off, the small interval called by the Pythagoreans τροφόν (see p.89 ) will be heard. We may easily obtain approximately the size of this interval on a stringed instrument by first playing an open string, then stopping off a Semitone by ear, and finally dividing by the eye the piece of string stopped off into three equal parts. If one (the first) of these parts is stopped off, the note will form with the open string, the interval in question sufficiently accurately for practical purposes. Since our natural Semitone is slightly larger than the Pythagorean Leimma (15/16 = 243/256 x 80/81.), 15 or 16 of the 256 parts would probably be taken from the whole string for sounding the Semitone by ear, instead of the 13 required by the Leimma. A third of these would therefore fall very near to the 5 forming the τροφόν . Like most of the Greek small intervals, this interval will seem very unharmonious. It will readily be admitted by most persons that the fifth of these small intervals might easily be regarded as the first to form an audible interval with the lowest note.

Let us now return to the 14th chapter.

Plutarch is discussing the double geometrical quaternion of the Timaeus. Here again are the consonant ratios exhibited.

Plut. De Anim. Procr. c. XIV, 1018 E:

ἔτι δὲ οὗτοι καὶ τοὺς τῶν συμφωνιῶν λόγους περιέχουσι ῥᾴδιον καταμαθεῖν. καὶ γὰρ διπλάσιος λόγος ἐστὶν ὁ τῶν δύο πρὸς τὸ ἕν, ἐν ᾧ τὸ διὰ πασῶν· καὶ ἡμιόλιος, ὁ πρὸς τὰ δύο τῶν τριῶν, ἐν ᾧ τὸ διὰ πέντε· καὶ ἐπίτριτος, ὁ πρὸς τὰ τρία τῶν τεσσάρων, ἐν ᾧ τὸ διὰ τεσσάρων· καὶ τριπλάσιος, ὁ πρὸς τὰ τρία τῶν ἐννέα, ἐν ᾧ τὸ διὰ πασῶν καὶ διὰ πέντε· καὶ τετραπλάσιος, ὁ πρὸς τὰ δύο τῶν η΄, ἐν ᾧ τὸ δὶς διὰ πασῶν· ἔνεστι δὲ καὶ ἐπόγδοος τῶν ὀκτὼ πρὸς τὰ ἐννέα, ἐν ᾧ τὸ τονιαῖον.

We meet with the arithmetical and harmonical means again in Chapter XV. The harmonic progression is so called because it expresses the 'first consonances,' as, the greater term to the least, the διὰ πασῶν, the greatest to the middle,

διὰ πέντε , the middle to the least διὰ τεσσάρων. The following correspondences are then given:

| | | |
|---|---|---|
| 12 | νήτη | } διὰ πέντε |
| 8 | μέση | |
| 6 | ὑπάτη | } διὰ τεσσάρων |

Chapter XVI gives arithmetical rules for finding the arithmetical and harmonical means of the duple and triple ratios of the Platonic quaternion under discussion. The terms are augmented by multiplication in order to allow of the insertion of the means, and again when the epitrite ratios (3:4) are filled up with epogdoa (8:9). The smallest number in this way becomes 384. This gives the following series for the first tetrachord:

| | Tone | Tone | Leimma | |
|---|---|---|---|---|
| 384 | 432 | 486 | | 512 |

in which the figures forming the ratio for the Leimma are double those given by Plato in the Timaeus ( i.e. 243:256). The larger numbers are necessary when the scale is continued to the Octave and the second Leimma appears. Theo Smyrnaeus discusses the two series at pp. 67 - 69 Ed. Hiller (De Mus. c. 14) See Boeckh, Kleine Schriften iii. Ueber die Bildung der Weltseele im Timaeos des Platon, p. 76, (158) fol.

In chapter XVII we have another statement of the ratios in which musical intervals are found.

Plut. De Anim. Procr. XVII, 1020 F. fol.:

ἐλήφθη διὰ τῶν ὀργάνων θεωρηθέν, ὅτι τὸ μὲν διὰ πασῶν τὸν διπλάσιον λόγον ἔχει, τὸ δὲ διὰ πέντε τὸν ἡμιόλιον, τὸ δὲ διὰ τεσσάρων τὸν ἐπίτριτον, ὁ δὲ τόνος τὸν ἐπόγδοον. ἔξεστι δὲ καὶ τὴν βασανίσαι τὸ ἀληθές, ἢ βάρη δυοῖν ἄνισα χορδῶν ἐξαρτήσαντας, ἢ δυοῖν ἰσοκοίλων αὐλῶν τὸν ἕτερον μήκει διπλάσιον τοῦ ἑτέρου ποιήσαντας.

Plutarch proceeds to illustrate the two methods (1021, A)

τῶν μὲν γὰρ αὐλῶν ὁ μείζων βαρύτερον φθέγξεται ὡς ὑπάτη πρὸς νήτην· τῶν δὲ χορδῶν ἡ τῷ διπλασίῳ κατατεινομένη βάρει τῆς ἑτέρας ὀξύτερον, ὡς νήτη πρὸς ὑπάτην· τοῦτο δ' ἐστὶ διὰ πασῶν.

The first statement is correct enough, not so the second.

As was pointed out at p. 52 the stretching weights must be as 1:4 ($= 1^2:2^2$), to produce the interval of an Octave (1:2)

The passage continues as follows:

Plut. De. Anim. Procr. XVII, 1021 A.

ὁμοίως δὲ καὶ τρία πρὸς δύο ληφθέντα μήκει καὶ βάρει τὸ διὰ πέντε ποιήσει. καὶ τέσσαρα πρὸς τρία τὸ διὰ τεσσάρων· ὧν τοῦτο μὲν ἐπίτριτον ἔχει λόγον, ἐκεῖνο δὲ ἡμιόλιον.

After showing that the Tone and its ratio are respectively the difference of the Fifth and Fourth and of their ratios and that the ratios in general may safely be regarded as representatives of the intervals in all operations, the chapter sums up with a statement of the consonant interval ratios. From the number of times this point is insisted upon, it would seem that the author did not consider it quite out of the domain of controversy.

Plut. De Anim. Procr. XVII, 1021 C:

φαίνεται τοίνυν ὅτι τὸ διὰ πασῶν τὸν διπλάσιον λόγον ἔχει, καὶ τὸ διὰ πέντε τὸν ἡμιόλιον, καὶ τὸ διὰ τεσσάρων τὸν ἐπίτριτον, καὶ ὁ τόνος τὸν ἐπόγδοον.

In Chapter xxii of the Dialogue De Musica, Plutarch shows how the Octave is divisible into two consonant intervals in two manners. The four numbers forming the Pythagorean 'harmony' (see p.   ) are used to illustrate the point.

12 νήτη διεζευγμένων

9 παραμέση

8 μέση

6 ὑπάτη μέσων

Plut. De Musica, xxii, 1138 E.:

τῆς γὰρ οὐχ ἁπλῶν ἐν Μουσικῇ συμφωνίας δύο διαστήματα μέσα εἶναι συμβέβηκεν ὧν τὴν ἀναλογίαν δείξομεν. ἡ μὲν γὰρ διὰ πασῶν ἐν διπλασίονι λόγῳ θεωρεῖται. ποιήσει δ' εἰκότως νόμον τὸν διπλάσιον λόγον κατ' ἀριθμὸν τὰ ἓξ καὶ τὰ δώδεκα· ἔστι δὲ τοῦτο τὸ διάστημα ἀπὸ ὑπάτης μέσων ἐπὶ νήτην διεζευγμένων. ὄντων οὖν τῶν ἓξ καὶ δώδεκα ἄκρων, ἔχει ἡ μὲν ὑπάτη μέσων τὸν τῶν ἓξ ἀριθμόν, ἡ δὲ νήτη διεζευγμένων τὸν τῶν δώδεκα. λαβεῖν δὴ λοιπὸν χρὴ πρὸς τούτοις ἀριθμοὺς τοὺς μεταξὺ πίπτοντας, ὧν οἱ ἄκροι ὁ μὲν ἐπίτριτος, ὁ δὲ ἡμιόλιος φανήσεται. εἰσὶ

ι τῶν ὀκτὼ καὶ τῶν ἐννέα· τῶν γὰρ ἓξ τὰ μὲν ὀκτὼ ἐπίτριτα, τὰ δὲ ἐννέα ἡμιόλια. τὸ μὲν ἓν ἄκρον τοιοῦτο· τὸ δ᾽ ἄλλο τὸ τῶν δώδεκα, τῶν μὲν ἐννέα ἐπίτριτα, τῶν δ᾽ ὀκτὼ ἡμιόλια. τούτων οὖν τῶν ἀριθμῶν ὄντων μεταξὺ τῶν ἓξ καὶ τῶν δώδεκα, καὶ τοῦ διὰ πασῶν διαστήματος ἐκ τοῦ διὰ τεσσάρων καὶ τοῦ διὰ πέντε συνεστῶτος, δῆλον ὅτι ἕξει ἡ μὲν μέση τὸν τῶν ὀκτὼ ἀριθμόν, ἡ δὲ παραμέση τὸν τῶν ἐννέα. τούτου γενομένου, ἕξει ἡ ὑπάτη πρὸς μέσην ὡς παραμέση πρὸς νήτην διεζευγμένων· ἀπὸ γὰρ ὑπάτης μέσων, διὰ τεσσάρων καὶ μέσην ἀπὸ δὲ παραμέσης ἐπὶ νήτην διεζευγμένων διὰ τεσσάρων. ἡ αὐτὴ δὲ ἀναλογία καὶ ἐπὶ τῶν ἀριθμῶν εὑρίσκεται· ὡς γὰρ ἔχει τὰ ἓξ πρὸς τὰ ὀκτώ, οὕτω τὰ ἐννέα πρὸς τὰ δώδεκα· καὶ ὡς ἔχει τὰ ἓξ πρὸς τὰ ἐννέα, οὕτω τὰ ὀκτὼ πρὸς τὰ δώδεκα· ἐπίτριτα γὰρ τὰ μὲν ὀκτὼ τῶν ἕξ, τὰ δὲ δώδεκα τῶν ἐννέα· ἡμιόλια δὲ τὰ μὲν ἐννέα τῶν ἕξ, τὰ δὲ δώδεκα τῶν ὀκτώ.

In the next chapter, xxiii, Aristotle's views on the 'harmony' are set forth.

Plut. De Mus. xxiii, 1139 C:

συνεστάναι δὲ αὐτὴν [illegible] ἔλεγεν ἐκ μερῶν ἀνομοίων, συμφωνούντων μέντοι πρὸς ἄλληλα· ἀλλὰ μὴν καὶ τὰς μεσότητας αὐτῆς κατὰ τὸν ἀριθμητικὸν λόγον συμφωνεῖν· τὴν γὰρ νήτην πρὸς τὴν ὑπάτην ἐκ διπλασίου λόγου ἡρμοσμένην τὴν διὰ πασῶν συμφωνίαν ἀποτελεῖν. ἔχει γάρ, ὡς προείπομεν, τὴν νήτην μονάδων δώδεκα, τὴν δὲ ὑπάτην ἕξ· τὴν δὲ παραμέσην συμφωνοῦσαν πρὸς ὑπάτην κατὰ ἡμιόλιον λόγον, ἐννέα μονάδων· τῆς δὲ μέσης ὀκτὼ εἶναι μονάδας ἐλέγομεν. ἀρκεῖσθαι δὲ διὰ τούτων τῆς Μουσικῆς τὰ κυριώτατα διαστήματα συμβαίνει τό τε διὰ τεσσάρων, ὅ ἐστι κατὰ τὸν ἐπίτριτον λόγον, καὶ τὸ διὰ πέντε, ὅ ἐστι κατὰ τὸν ἡμιόλιον λόγον, καὶ τὸ διὰ πασῶν, ὅ ἐστι κατὰ τὸν διπλάσιον.

x x x

( at the end of the chapter) 1139 F.

ἄλλως ἡ νήτη τῆς ὑπάτης διπλασία ἐστίν, ἡ δὲ παραμέση τῆς ὑπάτης ἡμιόλιος, ἡ δὲ μέση ἐπίτριτος πρὸς ὑπάτην ἥρμοσται.

See also Plutarch, Conviviales Disputationes, III., ix., Περὶ τοῦ ἢ πέντε πίνειν ἢ τρία, ἢ μὴ τέσσαρα. 678 end: καθάπερ γὰρ οἱ περὶ λύραν κανονικοὶ τῶν λόγων φασὶ τὸν μὲν ἡμιόλιον τὴν διὰ πέντε συμφωνίαν παρασχεῖν, τὸν δὲ διπλάσιον τὴν διὰ πασῶν, τὴν δὲ διὰ τεσσάρων ἀμυδροτάτην οὖσαν ἐν ἐπιτρίτῳ συνίστασθαι, οὕτως οἱ περὶ τὸν Διόνυσον ἁρμονικοὶ τρεῖς κατεῖδον οἴνου συμφωνίας πρὸς ὕδωρ, διὰ πέντε καὶ διὰ τριῶν καὶ διὰ τεσσάρων, οὕτω μὲν λέγοντες καὶ ᾄδοντες, πέντε πίνειν ἢ τρία, ἢ μὴ τέσσαρα.

Ptolemy is, of course, a thorough believer in the ratios. In explaining the Pythagorean doctrine he says,

Ptol. Harmonics I., v., p. 10, Ed. Wallis:

ἐφαρμόσαντες δὲ διὰ τοῦτο τοὺς ἐπιμορίους καὶ πολλαπλασίους λόγους ταῖς συμφωνίαις τὴν

καὶ διὰ πασῶν προσάπτουσι τῷ διπλασίῳ λόγῳ, τὴν δὲ διὰ πέντε τῷ ἡμιολίῳ, τὴν δὲ διὰ τεσσάρων τῷ ἐπιτρίτῳ.

In chapter viii Ptolemy describes the Monochord and gives directions for its use in proving the interval ratios. If the length of the string on one side of the movable bridge is to that on the other as 4 to 3, the sounds will make the consonance of the Fourth; if as 3 to 2, the Fifth; if as 2 to 1, the Octave; if as 8 to 3, the Eleventh; if as 3 to 1, the Twelfth; and if as 4 to 1, the Double Octave.

These results, it is stated, will be very exact.

Ptol. Harm. I., viii., p. 19, Ed. Wallis:

ἔπειτα, τοῦ τοιούτου κατειλημμένου καὶ διαλαμβανομένου τοῦ κανονίου τοῖς ἐκκειμένοις τῶν συμφωνιῶν λόγοις, εὑρήσομεν ἐκ τῆς ἐφ' ἑκάτερον τμῆμα τοῦ μαγαδίου παραγωγῆς, ὁμολογουμένης ταῖς ἀκοαῖς, ἐπὶ τὸ ἀκριβέστατον, τὰς τῶν οἰκείων φθόγγων διαφοράς.

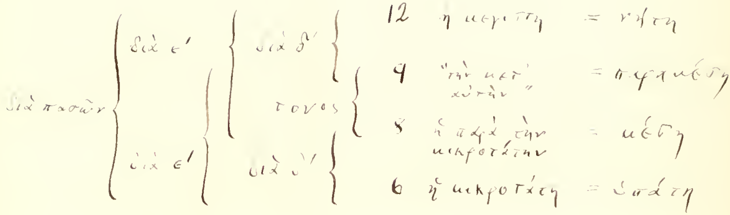
Ratios for four consonances, namely, the Fourth, Fifth, Octave, and Twelfth, are given by Dionysius Musicus. See Porphyrius ad Ptol. Harm. p. 219, Ed. Wallis. (Quoted by Westphal Rhythmik and Harmonik (= R.u.W.Metrik I.) Supplement p. 25.)

Iamblichus in his Life of Pythagoras describes the supposed visit of the philosopher to the smithy, where he first received the inspiration which led to the discovery of the ratios. On returning home Pythagoras stretched four strings with weights proportional to those which he had observed to sound the consonances.

XXVI (117) ἐν ᾧ δὴ γινομένων τῶν [illegible] καὶ χορδῶν ἐκκλίσεις συμφωνίας εὕρισκε τὰς προλεχθείσας ἄλλην ἐν ἄλλῃ συζυγίᾳ. τὴν μὲν γὰρ ὑπὸ τοῦ μεγίστου ἐξαρτήματος τεινομένην πρὸς τὴν ὑπὸ τοῦ μικροτάτου διὰ πασῶν φθεγγομένην κατελάμβανεν· ἦν δέ, ἡ μὲν δώδεκα τινῶν ὁλκῶν, ἡ δὲ ἕξ, ἐν διπλασίῳ δὲ λόγῳ ἀπέφαινε τὴν διὰ πασῶν, ὅπερ καὶ αὐτὰ τὰ βάρη ὑπέφαινε· τὴν δὲ τοῦ μεγίστου πρὸς τὴν παρὰ τὴν μικροτάτην, οὖσαν ὀκτὼ ὁλκῶν, διὰ πέντε συμφωνοῦσαν (ἡμιόλιον [illegible]), ἐν [illegible] λόγῳ

ὑπάρχειν ἐν ἡμιολίῳ λόγῳ, ἐν ᾧπερ καὶ αἱ
ὁλικαὶ ὑπῆρχον πρὸς ἀλλήλας· πρὸς δὲ τὴν
μετ' αὐτὴν μὲν τῷ ἄκρῳ, τοῖν δὲ δυοῖν
μείζονα, ἐννέα ἀριθμοὶ ὑπάρχουσαν, τὴν διὰ
τεσσάρων, ἀναλόγως τοῖς ὅροις, καὶ ταύτην
δὴ ἐπιτρίτου ἀκριβῶς καταλαμβάνεται,
ἡμιολίαν τὴν αὐτὴν φύσει ὑπάρχουσαν τῆς
μικροτάτης (118) τὰ γὰρ ἐννέα πρὸς τὰ ἕξ
οὕτως ἔχει, ὅνπερ τρόπον ἡ μεγίστη πρὸς τὴν
μετὰ τὴν μικράν, τὰ δὲ ὀκτὼ πρὸς μὲν τὴν τὰ ἕξ
ἔχουσαν ἐν ἐπιτρίτῳ λόγῳ ἦν, πρὸς δὲ τὴν ~~τὰ~~
δώδεκα ἐν ἡμιολίῳ. τὸ ἄρα μεταξὺ τῆς διὰ
πέντε καὶ τῆς διὰ τεσσάρων, ᾧ ὑπερέχει ἡ διὰ
πέντε τῆς διὰ τεσσάρων, ἐβεβαιοῦτο ἐν ~~ἡμι~~-
ἐπογδόῳ λόγῳ ὑπάρχειν, ἐν ᾧπερ τὰ ἐννέα
πρὸς τὰ ὀκτώ, ἑκατέρως τε ἡ διὰ πασῶν σύ-
στημα ἠλέγχετο, ἤτοι ἡ διὰ τεσσάρων καὶ
ἡ διὰ πέντε ἐν συναφῇ, ὡς ὁ διπλάσιος
λόγος ἡμιολίου τε καὶ ἐπιτρίτου, οἷον δώ-
δεκα, ὀκτώ, ἕξ, ἢ ἀναστρέφως τῆς διὰ τεσσά-
ρων καὶ τῆς διὰ πέντε, ὡς ὁ διπλάσιος
ἐπιτρίτου τε καὶ ἡμιολίου, οἷον δώδεκα
ἐννέα, ἕξ, ἐν τάξει τοιαύτῃ τῆς διὰ
πασῶν.

The following diagram may be useful in understanding the passage:




Bryennius Harmonica II. Sec. 1, p. 395, Ed. Wallis:

θεωρεῖται δὲ ἡ μὲν διὰ τεσσάρων συμφωνία ἐν λόγῳ ἐπιτρίτῳ, ἡ δὲ διὰ πέντε ἐν ἡμιολίῳ, ἡ δὲ διὰ πασῶν ἐν διπλασίονι, ἡ δὲ διὰ πασῶν καὶ διὰ τεσσάρων ἐν διπλασιεπιδιμερεῖ ἤτοι διπλασιοεπιδιτρίτῳ, ἡ δὲ πασῶν καὶ διὰ πέντε ἐν τριπλασίονι, ἡ δὲ δὶς διὰ πασῶν ἐν τετραπλασίονι.

From these passages it will be seen that among those most competent to judge there was substantial unanimity in the ancient world in regard to the ratios on which the Perfect Consonances depend. There can be not the shadow of a doubt

that these intervals, called διὰ πασῶν, διὰ πέντε, and διὰ τεσσάρων, are in fact the modern intervals of the Octave, Perfect Fifth, and Perfect Fourth. Even apart from the evidence of the mathematical ratios, it would not be very hard to establish their identity. Among other facts is that reported in the Aristotelian Problems XIX, 34 and 41, that the double διὰ πέντε and the double διὰ τεσσάρων are not consonant while the double διὰ πασῶν is consonant (see p. 79, and cp. Ptol. Harm. I., v., Ed. Wallis, p. 11 middle). But with the help of the ratios the proof is perfect. All three of these consonant intervals are bounded on both sides by discords. If the Octave is made a little too small or a little too large, the resultant discord becomes very painful, and the same holds true with slightly diminished force in the case of the Fifth and the Fourth. There is to be sure in modern music an interval which approaches the Fourth in point of size, called the Septimal Fourth or Subfourth, whose ratio is 16:21 ( it is the interval between the tonic or key-note and the dominant Seventh of the key, as C - F in C Major)—an interval which differs from a true Fourth (3:4) by the small distance expressed by the ratio 63:64, equal approximately to the fourth of a 'just'

semitone (15:16); but this interval, while frequently used in melody, rarely if ever appears as a member of a chord - that is to say, the notes composing it may occur successively, but not simultaneously, and the ancient consonances were undoubtedly tuned by simultaneously sounding the notes. See[1] Plut. De An. Proc. 1021 B.

Since, then, these intervals are bounded by the harshest of dissonances (harsher intervals are only to be found among very small intervals of the size of a semitone and less, which are the intervals next adjacent to Unison), it is inconceivable that any other intervals can be meant by the terms διὰ πασῶν, διὰ πέντε and διὰ τεσσάρων, or that any other intervals would have given such ratios, unless they were intervals to which the term σύμφωνος would have been inapplicable.

If, now, it may be stated as certain that the true interval ratios of the consonances were discovered by the ancients,

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1. The passage quoted at p.103 (1021 A) continues as follows: 1021 B:

ἐὰν δὲ ὡς ἐννέα πρὸς ὀκτὼ γένηται τῶν βαρῶν ἢ τῶν μηκῶν, ἡ ἀνισότης ποιήσει διάστημα τονιαῖον, οὐ σύμφωνον, ἀλλὰ ἐμμελὲς ὡς εἰπεῖν ἐμβραχύ τῷ

it followsthat the whole method of measuring intervals by means of their ratios may be regarded as firmly established for purposes of investigating the nature of ancient music.

The condition of affairs may be stated somewhat as follows: There are certain intervals in music of such a character that they produce a well defined and easily recognized pleasurable effect dependent on physical and physiological causes, which we assume of course to have remained unaltered through the ages. These intervals, if even slightly mistuned, are changed into intervals more discordant than any of the more usual intervals found in music.

Now it was discovered by the ancients that certain mathematical ratios were always associated with these intervals and it was also observed that, when these ratios were arranged on stretched strings and in other ways, the instrument always gave out the expected consonances. Small errors of observation undoubtedly existed. But it is possible to admit the existence

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τοὺς φθόγγους, ἂν κατὰ μέρος κρουσθῶσι, παρέχον-
ται φωνοῦντας καὶ προσηνεῖς· ἂν δὲ ὁμοῦ, τραχὺ καὶ λυπηρόν· ἐν ταῖς συμφωνίαις κἂν ὁμοῦ κρούωνται, κἂν ἐναλλάξ, ἡδέως προσίεται τὴν συνήχησιν ἡ αἴσθησις.

of inaccuracies and still to maint in that the value of the method is not impaired.[1] The errors cannot have been very large. For the ancients certainly dealt with intervals so small as sixtieths of a Fourth, which would be less than one-thirteenth of a just semitone. (See p.58 ).

If, then, it was possible for the ancients to attain a certain degree of accuracy in measuring three or four intervals whose character is such that their size is rigidly fixed and easily identified, we may safely affirm that other intervals, although less easy to tune correctly, must have been measured with the same accuracy, when tuned, and any variations in the measurements greater than those which may be attributed

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1. A.J.Ellis, in his translation of Helmholtz's Sensations of Tone, expresses the belief that the results obtained from the Monochord by the Greek mathematicians were "happy generalizations from necessarily imperfect instruments." Helmholtz-Ellis, Sensations of Tone, 2nd Eng. Ed. London, 1885, p.15, note.

to the imperfections of the measuring instrument must be the result of variations in the tuning. If it is reported by an ancient authority that a certain interval has a size denoted by a certain ratio, and the interval belonging to this ratio is produced mechanically by the best means at our command, we may be sure that we have reproduced the interval intended by the ancient musician, or that it differs from the correct interval by a small amount probably inappreciable. If the authorities differ materially as to size of the ratio which they assign to the interval between two given notes, it must be taken as evidence that the size of the interval was subject to variation, a state of affairs not unusual in homophonic or pure melodic music. But it must not be used as an argument that the whole matter of interval measurements was based on misapprehensions. The strangeness to our ears of any interval cannot condemn it. There are many instances in the music of existing non-European races of intervals that sound discordant to our ears.

It is for this reason that the consonant intervals alone with a few of the dissonant intervals, like the Tone and Leimma, derived from the consonances by simple subtraction, have been treated in these pages. The consonances, because they are

[illegible] definite

things or magnitudes, furnish the best means of testing methods of measurement. The chief interest in the dissonant intervals must lie in their place and function in the formation of scales, real and theoretical. The important and wide-reaching question of the Division of the Tetrachord, or the manner in which the two Fourths, which with a Tone compose almost every Octave scale, are divided, is omitted, being more germane to a study of the scales than of the intervals, as intervals.

The wonderfully large number of dissonant intervals claimed for Greek music by contemporary Greek musicians must therefore have really existed, and all attempts to translate the ancient notation into modern notation must take this fact into account. Our keyed instruments allow only thirteen notes to the Octave at Semitone distances. Only when the Greek notes happen to have pitches which can be represented satisfactorily ( it rarely happens that they can be represented accurately) by the notes of our key-board, is it possible to suppose that we have a fair representation of any given piece of Greek music. In other cases we must be satisfied to conclude that Greek music included elements foreign to our feeling.

It would not be surprising if future discoveries and researches should show that the Greeks were much more keenly alive to mere differences of pitch than are modern lovers of music. Trained as we are to bear the imperfections of tempered instruments, we are unable ( and it is even undesirable that we should try) to feel the deviations from the ideal intonation. In ancient Greek music, where Melody reigned supreme with Rhythm, there was abundant opportunity to use freely the pitch material at hand, and the evidence all tends to show that the opportunity was improved. But the modern world is richly compensated for any loss occasioned by our scantier use of pitch by the wonderful possibilities opened up by the very cause of our indifference to perfect tuning, namely, our Simultaneous Harmony.

# LIFE.

Charles William Leverett Johnson was born August 12, 1870 at Gambier, Knox Co., Ohio. He was prepared for college at Annapolis, Md., at the Boston Latin School, Boston, at the Perse School, Cambridge, England, and at Mr. Marston's School, Baltimore, Md. He entered the Johns Hopkins University as an Undergraduate Student in October, 1888, and received the degree of Bachelor of Arts, June, 1891. Since graduation he has attended the University as a Graduate Student in Greek, Latin, Sanskrit, and Comparative Philology. During the year 1893-94 he was Fellow in Greek, and is Fellow by Courtesy during the current year.

April, 1895.